Grand unification and the Weinberg–Salam model

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It is shown that the Weinberg–Salam model can be reproduced in the E_8 theory of grand unification. A nonrenormalized value of $\sin^2\theta_{W_0} = 0.3$ is predicted.

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The purpose of this paper is to draw attention to the possibility of reproducing the Weinberg–Salam model for a specified symmetric value of $\sin^2\theta_W = 0.3$ in the limits of the E_8 grand unification theory.

The agreement between the available experimental data and the predictions of the Weinberg–Salam model with $\sin^2\theta_W = 0.23$ is so impressive that it would be surprising if a deep theoretical principle allowing calculation of this value of $\sin^2\theta_W$ did not exist.

One of the possible ways to determine the value of $\sin^2\theta_W$ is based on grand unification of strong, weak, and electromagnetic interactions. The gauge theories of grand unification are built on the basis of the grand group G which describes the symmetry between the quarks and the leptons and which contains the gauge groups of strong, weak, and electromagnetic interactions. The manner in which the latter are embedded in G determines the value of $\sin^2\theta_{W_0}$. This symmetric value, which corresponds to the region of (superhigh) energies where the original grand-symmetry breaking can be disregarded, can be renormalized for the present-day energies. In the well-known SU(5)-, SO(10)-, and E_6 theories[1] $\sin^2\theta_W = 3/8$ and after renormalization it assumes the necessary value. In the indicated schemes for the description of the fundamental fermions, however, it is necessary to use the cited grand-group representations, which is unsatisfactory. The most consistent in this scheme are the exceptional E_7 and E_8 models. First let us examine the more economical E_7 scheme. The fundamental fermions and gauge fields in this theory are contained in the $\underline{56}$ and $\underline{133}$ multiplets, respectively. The expansions of these representations in the subgroup $SU(6)_{\text{flavor}} \otimes SU^c(3)$ have the form

$$\underline{56} = (20.1^C) + (6.3^C) + (\bar{6}.\bar{3}^C) \tag{1}$$

$$\underline{133} = (35.1^C) + (\overline{15}.3^C) + (15.\bar{3}^C) + (1.8^C)$$

and for the standard definition of the electric charge $Q = \text{diag}\,(2, 3, -1/3, -1/3, 2/3, -1/3, -1/3)$, $\sin^2\theta_W$ assumes a too large value of 3/4 (2/3 after renormalization[4]).

In the framework of the E_7 group, however, there is a second possible definition of a charge[2]

$$Q = \text{diag}\,(2/3, -1/3, 2/3, 2/3, -1/3, -4/3). \tag{2}$$

Our first remark is that definition (2) gives $\sin^2\theta_W = 0.3$ (2/9 after renormalization). Moreover, in contrast to the latest case, charged leptonic singlets appear under the conventionally defined[2] $SU(2)_W$ weak-interaction group, which makes it possible to fulfill the Weinberg-Salam prescription for $e_{\bar{R}}$ and $\mu_{\bar{R}}$. Unfortunately, as seen in Eq. (2), for such a definition the theory has no quark $SU(2)_W$ singlets with a charge $-1/3$, i.e., d_R must be embedded in the doublet, which contradicts the available data.[5] Our main observation, therefore, is that the indicated difficulty can be eliminated by going over to E_8 as the grand group. In fact, the fermions, like the gauge fields, are transformed in the E_8 scheme according to the $\underline{248}$ representation (this allows a natural introduction of supersymmetry in the theory),[3] whose expansion in the maximal subgroup $E_7 \otimes SU(2)$ is as follows:

$$\underline{248} = (56,2) + (133,1) + (1,3). \tag{3}$$

Keeping the standard definition of the weak-interaction group $SU(2)_W < SU(6) < E_7 < E_8$ and the definition of the charge (2), we are convinced that $\sin^2\theta_W = 0.3$. It can be seen in Eqs. (1) and (3) that the color-triplet quarks are contained in the theory in the $(6.2) + (\overline{15}.1)$ group representation $[SU(6) \otimes SU(2)]_{\text{flavor}}$ and since the $\underline{15}$ multiplet contains two $SU(2)_W$ singlets with a charge $-1/3$, there is no difficulty with the right-handed d quark. Therefore, in the limits of the E_8 theory the quantum numbers of the known fermions can be fully reconciled with the Weinberg–Salam prescription. The value of $\sin^2\theta_{W_0}$ is close to that required by the experiment. The renormalized value of $\sin^2\theta_W$ here depends on the specific spontaneous violation. Since the theory has 27 quark flavors plus 8 quarks with the quantum numbers of gluons (see Ref. 6), some of them must acquire superheavy masses to ensure asymptotic freedom of quantum chromodynamics, and the value of $\sin^2\theta_W$ depends on which quarks in the theory remain light.

Having the highest rank of all the exceptional groups ($E_8 \supset E_7 \supset E_6 \supset F_4 \supset G_2$), the E_8 group in this sense realizes the extremal possibility of the grand symmetry. However, in spite of its abundance of fundamental fermions, the E_8 theory, because of the singularity indicated in this paper, deserves a comprehensive investigation.

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