Allowance for the antiquark "sea" in determination of $\sin^2\theta_W$ from the total neutrino cross sections

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(Submitted 4 July 1979)

Pis'ma Zh. Eksp. Teor. Fiz. **30**, No. 6, 364–369 (20 September 1979)

Simple formulas are obtained and estimates of the contribution from the antiquark "sea" are given for the expressions that connect the ratios of the neutrino NC and CC cross sections with the parameter $\sin^2\theta_W$ of the Weinberg–Salam model.

PACS numbers: 13.15. + g, 12.30.Cx, 12.40.Cc

The level of accuracy of the latest experiments on inclusive neutrino processes with neutral (NC) and charged (CC) currents is such that a quantitative comparison of their results with theoretical predictions requires taking into account the nucleon structure in the quark-parton model. In particular, if a more accurate value of the Weinberg angle can be determined from the ratio of the total cross sections $R^{\overset{(-)}{\nu}} = \sigma^{\overset{(-)}{\nu}}_{NC}/\sigma^{\overset{(-)}{\nu}}_{CC}$, then the contribution of the antiquark "sea" in the nucleon must be taken into account.[1] It is known that in the approximation of the absence of such a contribution the parameter $\sin^2\theta_W$ (θ_W is the Weinberg angle) can be determined separately and collectively for the ν and $\bar{\nu}$ processes:

$$R^{\nu} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W, \tag{1a}$$

$$R^{\bar{\nu}} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{9}\sin^4\theta_W, \tag{1b}$$

$$2\sin^2\theta_W = 1 - 3R^{\nu} + R^{\bar{\nu}}. \tag{1c}$$

Note that the condition for valid solutions of $\sin^2\theta_W$ in Eqs. (1a) and (1b) imposes constraints on the possible values of R^{ν} and $R^{\bar{\nu}}$. We can easily see, for example, that the condition $R^{\bar{\nu}} \geqslant 0.3875$ must be satisfied. This relation must satisfy the results of experiments only at low energies[1)] when Eqs. (1) can be used. Allowance for the "sea" gives rise to the appearance of correction terms in Eqs. (1), which contain new parameters associated with the ratio of the average distribution widths of antiquarks and quarks in the proton in the Bjorken scaling variable x. Below we shall use two types of such parameters:

$$\kappa = \frac{\int x[\bar{u}(x) + \bar{d}(x)]dx}{\int x[u(x) + d(x)]dx}, \quad \kappa_s = \frac{\int x\bar{s}(x)dx}{\int x[u(x) + d(x)]dx}; \tag{2a}$$

TABLE I.

Group	R^{ν}	$R^{\bar{\nu}}$	$\sin^2\theta_W$ according to Eq. (3b) at $\eta_s = 0$			
			$\eta = 0$	$\eta = 0,10$	$\eta = 0,15$	$\eta = 0,20$
CITF [3]	0,27 ±0,02	0,40 ±0,08	0.295 ±0,05	0,32 ±0,07	0,34 ±0,08	0,36 ±0,09
HPWF [4]	0,30 ±0,04	0,33 ±0,09	0,215 ±0,075	0,22 ±0,09	0,23 ±0,11	0,23 ±0,12
BEBC [5]	0,33 ±0,05	0,36 ±0,07	0,185 ±0,08	0,19 ±0.10	0,20 ±0.11	0,20 ±0,12
CDHS [6]	0,295 ±0,01	0,34 ±0,03	0,23 ±0,02	0,24 ±0,03	0,24 ±0,03	0,25 ±0,04

$$\eta = \frac{\int x[\bar{u}(x) + \bar{d}(x) + \bar{s}(x)]dx}{\int x[u(x) + d(x) + s(x)]dx}, \quad \eta_s = \frac{\int x\bar{s}(x)dx}{\int x[u(x) + d(x) + s(x)]dx}. \tag{2b}$$

Here $u(x)$, $\bar{u}(x)$, $d(x)$, $\bar{d}(x)$, and $s(x) = \bar{s}(x)$ are functions of the above-mentioned distributions of the u, d, and s quarks and antiquarks, respectively. The general expressions for the dependence of R^{ν} and $R^{\bar{\nu}}$ on $\sin^2\theta_W$ and on parameters such as (2) are fairly cumbersome.[1] It is easy to obtain, however, the following simple formulas for $\sin^2\theta_W$, which are analogous to Eq. (1c)[2]:

$$2\sin^2\theta_W = 1 - 3R^{\nu}\frac{(1+\kappa/3)}{(1-\kappa)} + R^{\bar{\nu}}\frac{(1+3\kappa)}{(1-\kappa)} + 6\kappa_s\frac{(R^{\bar{\nu}} - R^{\nu})}{(1-\kappa)}, \tag{3a}$$

$$2\sin^2\theta_W = 1 - 3R^{\nu}\frac{(1+\eta/3)}{(1-\eta)} + R^{\bar{\nu}}\frac{(1+3\eta)}{(1-\eta)} + 2\eta_s\frac{(R^{\bar{\nu}} - R^{\nu})}{(1-\eta)}. \tag{3b}$$

Table I gives the values of $\sin^2\theta_W$, which were calculated for four values of the parameter η according to Eq. (3b) for the quantities R^{ν} and $R^{\bar{\nu}}$ measured in high-energy neutrino experiments. Note that allowance for the sea of strange quark-antiquark pairs at $\eta_s = 0.015$ adds a positive component to $\sin^2\theta_W$ [the last term in Eq. (3b)], which does not exceed 0.003.

The results of the CDHS group for R^{ν} and $R^{\bar{\nu}}$ pertain to the Fe target.

TABLE II. Lower boundaries of the parameter κ.

$R^{\bar{\nu}}$	$\kappa_s = 0$	$\kappa_s = 0.01$	$\kappa_s = 0.02$	$\kappa_s = 0.03$	$\kappa_s = 0.04$
0,33	0,34	0,31	0,28	0,26	0,23
0,34	0,27+	0,24	0,21	0,18	0,15
0,35	0,20	0,17	0,14	0,11	0,07
0,36	0,14	0,11	0,07	0,04	0,03

Note: + If $R^{\bar{\nu}} = 0.34$ pertains (as in the case of the data of the CDHS group) to the Fe target, then allowance for nonisoscalarity decreases the lower bound: $\kappa \geqslant 0.24$.

Allowance for nonisoscalarity corresponds to addition of an extra term $-\delta[R^{\nu}(3-\eta)+R^{\bar{\nu}}(1-3\eta)]/(1-\eta)$ on the right-hand side of Eq. (3b) and of the factor $(1-\delta/3)$ on the left-hand side, where $\delta=(A-2Z)/3A$ (A is the atomic weight and Z is the number of the element; for Fe $\delta=0.023$). This reduces in Table I the values of $\sin^2\theta_W$ for the CDHS data by ≈ 0.01.

We note that the values of $\sin^2\theta_W$ in Table I are almost the same as those for $\sin^2\theta_W$, determined by the indicated groups using the best description of the experimental distributions of inclusive NC events with respect to the energy transfer to the hadrons E_h or according to the scaling variable $y=E_h/E_\nu$.

The condition for valid solutions for $\sin^2\theta_W$ in the equation analogous to relation (1b) when the sea is taken into account leads to the following approximate inequality[3)] which is sensitive to κ_s:

$$\beta \gtrsim \frac{1-2R^{\bar{\nu}}}{(2R^{\bar{\nu}}-0.1)+\lambda(2.3R^{\bar{\nu}}-0.3)}, \quad \beta = \frac{1+3\kappa}{3+\kappa}, \quad \lambda = \frac{8\kappa_s}{1+3\kappa}. \qquad (4)$$

Table II gives lower limits for the parameter κ, which were determined from an exact inequality similar to (4) for different values of κ_s.

Thus, the available data for $R^{\bar{\nu}}$ indicate that the contribution of the sea to high-energy antineutrino interactions ($\sim$20–200 GeV) are sufficiently large and determine the lower limit of this value. We remind that the data for different neutrino experiments correspond to the following average parameters[7]: $\kappa \approx 0.15 \pm 0.02$ and $\kappa_s \approx 0.02 \pm 0.01$. We also remind that a more accurate experimental determination of the values of $\sin^2\theta_W$, κ, and κ_s generally requires taking into account the effects associated with the nonscaling dependence of the distribution functions of quarks and antiquarks on the square of the 4-momentum transfer to the nucleon and radiative corrections.[8,9] Therefore, it is desirable to determine the indicated parameters from the integral characteristics (for example, from the total cross sections) in which these effects are missing or manifest themselves minimally, rather than from the differential distributions.

TABLE III.

Group, R^{ν} and $R^{\bar{\nu}}$	η_s	0	0,01	0,02	0,03	0.04
HPWF [4]	η	0,46	0.43	0,405	0,38	0,34
0,30, 0,33	z	0.27	0,26	0,255	0,25	0,245
BEBC [5]	η	0.27	0,23	0,19	0,15	0,10
0,33 0,36	z	0,21	0.20	0.20	0.20	0,19
CDHS [6]	η	0,34	0,32	0,29	0,27	0,245
0,295, 0,34	z	0,26	0,26	0,255	0,25	0,25

Note: The corrections due to nonisoscalarity of the target were taken into account in determining η and z from the CDHS data. The results obtained from the CITF data correspond to anomalously large values: $\eta \approx 0.6$ and $z \approx 0.7$.

If the expressions analogous to Eqs. (1a) and (1b) are written in explicit form when the sea is taken into account, then we can obtain from them one more equation for the coupling $z \equiv \sin^2\theta_W$ with R^{ν} and $R^{\bar{\nu}}$, which is independent of Eq. (3):

$$\frac{5}{9} z^2 (1-\gamma)(1+\gamma-\frac{3}{5}\epsilon) + \frac{1}{6} z(1-\gamma)\epsilon - [(R^{\bar{\nu}}-R^{\nu})\gamma + \frac{1}{4}(R^{\bar{\nu}}-R^{\nu}\gamma)\epsilon] = 0,$$

$$\epsilon = \eta_s (3-\gamma), \quad \gamma = (1+3\eta)/(3+\eta). \tag{5}$$

(An analogous expression couples z with β.) Eliminating the variable z from Eqs. (3) and (5) and ignoring, for simplicity, terms incorporating the strange sea, we obtain the following relation for determining a single value of η, which satisfies both the ν and $\bar{\nu}$ data:

$$[(0,5 - R^{\nu}) - \gamma(0,5 - R^{\bar{\nu}})]^2 = \frac{9}{5}\gamma \frac{(1-\gamma)}{(1+\gamma)}(R^{\bar{\nu}} - R^{\nu}), \quad \eta = \frac{3\gamma - 1}{3-\gamma}. \tag{6}$$

(These equalities remain valid after the substitution of $\gamma \to \beta$ and $\eta \to \kappa$.) The data for R^{ν} and $R^{\bar{\nu}}$ obtained in high-energy experiments determine, according to equations such as (6) which take into account terms with η_s, the following solutions for the parameters η and, according to Eq. (3b), the solutions for $z = \sin^2\theta_W$ (see Table III).

In evaluating the results of their large statistics experiment,[6] the CDHS group used the parameters $\eta \approx 0.07$ and $\xi_s = \int 2sxdx / \int x(u+d)\,dx = 0.03$, which corresponds to $\eta_s = 0.0148$. According to the γ distributions, the authors obtained z equal to 0.243 ± 0.021 and 0.21 ± 0.09 for ν and $\bar{\nu}$ events, respectively; the average value of $\sin^2\theta_W$ for all the data is 0.24 ± 0.02. If we take $\eta_s = 0.0148$, determine η and z from the above-mentioned exact relation such as (6), and allow for the corrections for nonisoscalarity, then we get $\eta = 0.305$ and $\sin^2\theta_W = 0.256$.

Thus, according to Table III, the combined results of measurement of the total NC and CC cross sections of $\nu_\mu N$ and $\bar{\nu}_\mu N$ interactions indicate that the contribution from the sea antiquarks in the high-energy region is large for any reasonable assumptions about the effect produced by the strange sea. Here we should mention that the values of n and z cited in Table III cannot be considered sufficiently dependable. This is attributable to the fact that, as seen in Eq. (6), the existing measurement error of R^{ν} and $R^{\bar{\nu}}$ produces a large error in the $(R^{\bar{\nu}} - R^{\nu})$ factor. Thus, even the most accurate experiment of the CDHS group gives $(R^{\bar{\nu}} - R^{\nu}) = 0.045 \pm 0.032$.

In conclusion, we note that apparently the most reliable method of determining $\sin^2\theta_W$, which has low sensitivity to experimental corrections, is the extraction of this parameter from the Paschos–Wolfenstein relation.[10] As follows from its derivation by the authors and as can easily be verified, this relation does not depend on the contribution from the sea; it includes only a correction for nonisoscalarity of the target:

$$(\sigma^{\nu}_{NC} - \sigma^{\bar{\nu}}_{NC}) / (\sigma^{\nu}_{CC} - \sigma^{\bar{\nu}}_{CC}) = \frac{1}{2}(1 - 2\delta) - (1 - \frac{7}{3}\delta)\sin^2\theta_W. \tag{7}$$

The author is grateful to L. B. Okun' for stimulating the writing of this paper and to V. I. Zakharov for reading the manuscript.

1) At $E_{\bar{\nu}} \leqslant 10$ GeV the GGM group obtained $R^{\bar{\nu}} = 0.39 \pm 0.06$.[2]

2) As usual, all the relations pertain to the case of an isoscalar target.

3) For $\kappa \geqslant 0.15$ the error in calculating $\kappa_{\min}$ from inequality (4) does not exceed 5%; the approximate inequality such as (4) for η is more cumbersome.

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