# Few-nucleon correlations in nuclei and cumulative-particle yields

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The expressions for the single inclusive spectra of cumulative $p$, $\pi$, $K$...$H^{2,3}$, $He^3$...were obtained by using the single-particle density matrix of the nucleons in the nucleus. The dependence of the spectra on the atomic number and on the momentum of the cumulative particle is explained.

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The yield of the cumulative $p$ and $\pi$ particles in the reactions $p+D(\mathrm{He}^4)\rightarrow p,\pi+X$ [2] was calculated (in accordance with the experiment) by Frankfurt and Strikman[1] in the framework of the standard Gribov-Glauber picture for scattering of hadrons by nuclei. For nuclei with $A\leqslant 6$ it was sufficient to confine oneself to the pair correlation approximation. For nuclei with $A\geqslant 12$ a neglect of the triple (quadruple) correlations is too crude an approximation. The formulas obtained in this paper take into account the scattering by any few-nucleon correlations (FNC) and relate the spectra of all the cumulative particles.

Let us introduce a single-nucleon density matrix $\rho_A^{p(n)}(\alpha,k_\perp)$-the probability of finding a proton (neutron) in the nucleus, which carries the transverse momentum $k_\perp$, and a fraction of the momentum of the nucleus $\alpha/A$ in the system, where the nucleus is fast.[1)] $\rho_A^N$ is normalized according to

$$\int_0^A \rho P_A^{(n)} (a, k_\perp) \frac{da}{a} d^2 k_\perp = z(A - z).$$

In the absence of enhanced Reggeon diagrams and cascades the contribution of the spectator mechanism (which is the dominating mechanism at small $k_\perp$ [1,3]) has the form[2]

$$G_h^{A/N} (a, k_\perp) \equiv \frac{a\, d\sigma^{h+A\to N+X}}{d a\, d^2 k_\perp} = \kappa_h \rho_A^N (a, k_\perp) \sigma_{in}^{hN}. \tag{1}$$

Equation (1) was derived on the basis of the observation that, because of the topological structure of the AGK rules,[4] the contribution from the scattering by nucleons not belonging to the FNC is reduced as a result of calculation of the inclusive spectrum.[3] Allowance for the direct mechanism [compare Eq. (7)] effectively increases $\kappa_h$ by a factor of 20 to 40%. A calculation of the screening factor in case of a pair correlation shows[1] that at $2 > \alpha > 1.3$ $\kappa_h$ is almost independent of $\alpha$ and $k_\perp$ and depends weakly on the incident hadron ($\kappa_\pi \approx 0.65$, $\kappa_p \approx 0.55$). To effectively destroy a more complex correlation, all the nucleons correlated with the data must be knocked out (see Ref. 3). The estimates show that this process is of the same order of smallness as $\kappa_h$ for the pair correlation.

The scattering of fast secondary particles in the heavy nuclei may lead to an additional destruction of few-nucleon, short-range correlations by a factor of $\lambda_A$. It is clear that $\lambda_A$ is proportional to the number of nucleons in a tube of area $\pi\rho_h^2$, where $\rho_h$ is the radius of the strong interaction. In the framework of the two-component model[5] $\lambda_A \approx 1$ for $A = 12$ and $\lambda_A \approx A^{0.2}$ in the range $12 < A < 200$.[4] Below we shall use this estimate of $\lambda_A$ for comparison with experiment (in principle, $\lambda_A$ can be measured by an independent method[1,3]). As a result, by disregarding the evaporative contribution (which is justifiable at $\alpha > 1.3$), we get

$$G_h^{A/N} (a, p_\perp) = \rho_A^N (a, p_\perp) \sigma_{inel}^{hN} \lambda_A \kappa_h. \tag{2}$$

Since $\kappa_h$ depends much more weakly on $\alpha$ and $k_\perp$ than does $\rho_A^N$ (see Ref. 1), the measurement of the spectra of cumulative nucleons at high energy is almost equivalent to measurement of the single-nucleon density matrix of the nucleus [(it is easy to show that Eq. (2) is also valid for fragmentation of D and He in the $D(He^4) + A \to p + X$ reactions; moreover, $\kappa_A \sim A^{-1/3}$, which agrees well with experiment[6] but contradicts the empirical hypothesis of nuclear scaling just like the data for scattering of $\gamma$ and $\bar{\nu}$ by nuclei (see Ref. 1)].

It is useful to expand $\rho_A^N$ in a series in $r$-nucleon correlations:

$$\rho_A^N (a, k_\perp) = A \sum_{r=2}^{A} a_r (A) \rho_r (a, k_\perp), \text{where } \rho_r (a, k_\perp) = 0 \text{ when } a > r. \tag{3}$$

For intermediate and heavy nuclei the $A$ dependence of the coefficients can be evaluated according to Eq. (4)[5]:

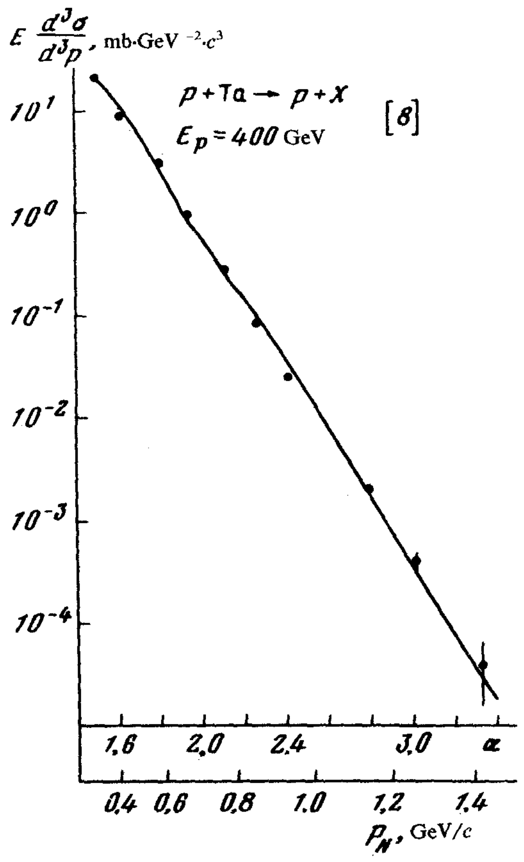



FIG. 1. Yield of cumulative protons in the $p+\mathrm{Ta}\rightarrow p+X$ reaction at $E_p=400$ GeV.[8] The data at $\theta=160°$ are extrapolated to $\theta=180°$, $n=2.8$, $\alpha_3/\alpha_2=1.3$, $\alpha_4/\alpha_2=1/10$, $\alpha_5/\alpha_2=1/30$, and $\alpha_6/\alpha_2=1/100$.

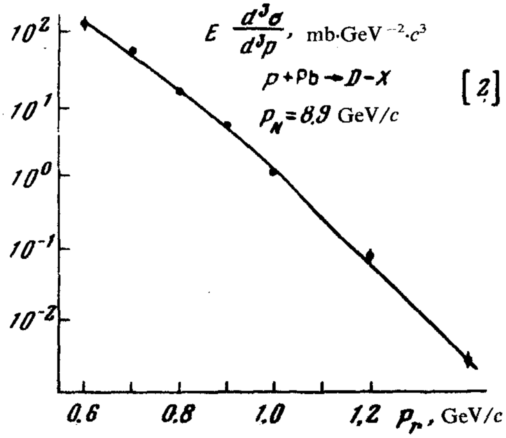
$$a_r(A)\sim\frac{1}{A}\int[\rho_A(r)]^r d^3r, \text{ where } \rho_A(r) \tag{4}$$

$$\int\rho_A(r)\,d^3r.$$

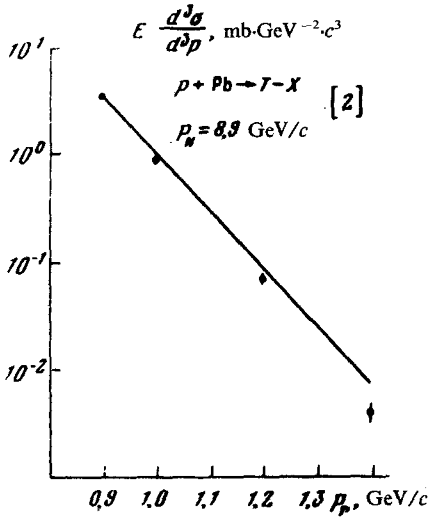
For $12<A<200$ $\alpha_2\sim A^{0.15}$, $\alpha_3\sim A^{0.22}$, and $\alpha_4\sim A^{0.27}$. A theoretical calculation[7] gives $\alpha_2\approx 3$ for $He^4$. An analysis of data for photodisintegration and $\pi$ capture indicates that $\alpha_2$ $(C^{12})\approx 8$ (see Ref. 1) (here we used the normalization $\rho_2(\alpha,k_\perp)\approx\rho_D(\alpha,k_\perp)$.

The calculations presented above show that in the absence of cascades $G_h^A\rightarrow p$ $A^{1,2}$, because the triple and pair correlations give approximately the same $A$ dependence. The cascade from the fast particles accelerates the dependence on $A$ by a factor of $A^{0.2-0.3}$ without substantially changing the shape of the spectrum. Finally, $G^{A\rightarrow p}$ $\sim A^{1.4-1.5}$, in agreement with the experiment.[8] For example, using the model[5] we get $G^{\mathrm{Ta}\rightarrow p/C^{12}\rightarrow p}\approx 3.6$ (the value obtained in the experiment[8] is 3.7).

To estimate the functional dependence of the contributions from many-nucleon correlations to $\rho_A^N$ we must assume that the wave function of the $r$-nucleon correlation is equal to the convolution of $r-1$ wave functions of the pair correlation. Thus, assuming that $\rho_2(\alpha_1 k_\perp)\sim(2-\alpha)^n$ and using a method similar to the derivation of the




a




b

FIG. 2a and 2b. Calculation of the spectra of cumulative D and T at $\theta = 180°$ and $E_p = 9$ GeV[2] according to Eq. (8).

relations for dimensional calculation of nuclei,[9] we obtain (for simplicity, we confine ourselves to the case $k_\perp - 0$):

$$\rho_A^N(a, k_\perp) = \sum_{r=2} a_r(A) \left[\frac{r-a}{r-1}\right]^{n(r-1)+r-2} \tag{5}$$

In Eq. (5) $a_r/r$ is proportional to the probability of $r$-nucleon correlation. It follows from the data[2] for the $p + D \to p + X$ reaction that $n = 2.8 \pm 0.1$.[6] Figure 1 compares the data[8] for the $p + Ta \to p + X$ reaction at $E_p = 400$ GeV with Eqs. (2), (5), $(a_2/3)/(a_2/2) = 0.2$, and $(a_4/4)/(a_2/2) \sim 0.05$. Thus, Eqs. (2)–(5) make it possible to describe the dependence of the cumulative protons on the $A$ and $a$ spectrum at high energy. The same calculation shows that the average transverse momentum of the cumulative protons must increase substantially with increasing $\alpha$. This also corresponds to the data.[8]

The effective number of collisions decreases with decreasing initial energy; moreover, the $r$-fold correlation contributes only in the region $\alpha < \alpha_{\max}$ $(r, E_h)$. If we assume light-cone scaling[3] at low energies (1 GeV), where there is effectively only one collision then Eq. (2) in this region becomes

$$G_h^{A/p}(a, k_\perp) = \sigma_{inel}^{hA} \sum_r \rho_r(a, k_\perp)\, \theta[a_{max}(r, E_h) - a]. \tag{6}$$

At $T_p = 640$ MeV a calculation according to Eq. (6) with the same $\alpha_{4,3}/\alpha_2$ ratio gives a slope $B \sim 17$ GeV$^{-2}$ $[G_p \sim \exp(-B\,^2_p)]$ in the region of the threefold-fivefold correlations. Experimentally[10] $B = 17$–$20$ GeV$^{-2}$. Equation (6) also predicts that $G_p^{A/p}\, \sigma_{in}^{hA}\, a_{3,4} \sim A^{0.7} A_{0.3} = A$, which agrees quite well with experiment at $A > 12$. The increase with $A$ of the contribution of the many-nucleon correlations decreases $B$ (in agreement with experiment[10]) (an increase of the $A$ dependence of the spectrum in the energy region of 1 to 10 GeV was accounted for by Frankfurt and Strikman[1]). The jumps at the boundary of the 3, 4... correlations, which formally follow from Eq. (6), must be masked extensively by the motion of the correlations in the average field of the nucleus and by the interaction in the final state, which is large at low energies.

Thus, we have shown that the FNC hypothesis can uniquely describe the data for production of cumulative protons in the energy range of 0.6 to 400 GeV.

The production cross section of cumulative pions ($K$, $\Lambda$) does not contain the Glauber screening factor[1] and, because of kinematics, the contribution from low-energy cascades in this case apparently is small:

$$G_h^{N\to\pi}(X, p_\perp) = \int \sum_{p,n} \rho_A^N(a, k_\perp) \frac{da}{a}\, d^2k_\perp G_h^{N\to\pi}(X/a, p_\perp - x/a\, k_\perp). \tag{7}$$

Using Eq. (7) and parametrizing the experimental data (8) in the form $G_p^{A/p} \sim \rho_A^N \exp(-7.2\alpha)$; $G^{N\to\pi} \sim (1-\alpha)^n$, $n \sim 2-3$, we can calculate without free parameters the momentum dependence of the spectrum of the cumulative pions. We have $G_h^{A/\pi} \sim \exp(-7.2\alpha_\pi)/\alpha_\pi^n \sim \exp(-T_\pi/T_0)$, where $T_0 \approx 65$ MeV, in good agreement with Ref. 2. The $A$ dependence of the spectrum is determined by the behavior of $\alpha_{2,3}$; i.e., it is expected that at $\alpha > 1$ $G_b^{A/\pi} A^{1.2}$. A slightly slower increase of $G_p^{A/\pi}$ at $E_p \sim 10$ GeV ($A^{1.1}$) is apparently attributable to the initial low energy (see Ref. 3).

In the picture being considered the light cumulative nuclei (D,T) are produced as a result of adhesion of cumulative protons and neutrons with similar $z$ in the field of the nucleus (this mechanism is similar to that in Ref. 1). The production cross section of a $B$ nucleus from $Z$ protons and $N$ neutrons at $p_B > p_F B$ and small $p_{B_\perp}$ is

$$G_p^{A/B}(P) = \xi_B [G_p^{A/p}(p/B)]^Z [G_p^{A/n}(p/B)]^N (\lambda_A \sigma_{in}^{hN} A p^2/p_F^z)^{1-B}$$
$$\approx \xi_B \left(\frac{N}{Z}\right)^N [G_p^{A/p}(p/B)]^B (\lambda_A \sigma_{in}^{hN} A p^2/p_F^2)^{1-B}. \tag{8}$$

The $p^{-2(B-1)}$ factor takes into account the small adhesion of the nucleons. $\xi \sim 10^{-2}$ can be theoretically estimated in terms of standard nuclear theory. Equation (8) describes the momentum dependence of the cumulative nucleus (see Fig. 2) and the $A$ dependence of the spectra well.[2,8,10]

Thus, all the available data for single-inclusive spectra are in agreement with the developed approach. Correlation measurements are required for a final determination of the role of cascades and of the effect of transmission of the cumulative particles through the nucleus. The data of the first neutrino experiments[12] are in agreement with the correlation between the average $x$ and $\alpha$ predicted in Ref. 3.

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[1)]We disregard here the possible admixture of resonances ($\Delta, N^*$) in the wave function of the nucleus. This contribution practically does not change the final result.

[2)]$\alpha_b$ is connected with the lab. momentum of the cumulative particle "$b$" according to $\alpha_b = (\sqrt{m_b^2 + p_b^2} - p_{b_z})/m_N$.

[3)]In Ref. 1 Eq. (1) was proved in the pair-correlation approximation.

[4)]We are grateful to Nikolaev for discussing the parameters of the model.[5]

[5)]We are grateful to V.A. Khodel for explaining how Eq. (4) originates in the theory of Fermi liquid.

[6)]Note that the quark calculation gives $n = 5$.[9]

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