Calculation of the meson-meson interaction cross section in quantum chromodynamics

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The total cross section of interaction of ρ mesons at high energies is calculated in quantum chromodynamics in lowest-order perturbation theory. The result is in agreement with predictions of the additive quark model.

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At present, quantum chromodynamics (QCD) is the most popular field model of strong interactions. The "asymptotic freedom" allows the use of perturbation theory for calculation of the amplitudes of a number of processes occurring at small distances.[1] In a previous paper Balitskiĭ and Lipatov[2] attempted to describe by using QCD the process of high-energy scattering of charmed particles. In this paper this description is generalized to the case of particles composed of light quarks.

We use the idea of duality[3,4] to show that vacuum polarization by light quarks $\Pi(q^2)$, which is a dispersion integral of the e^+e^- annihilation to hadrons cross section, can be calculated in QCD in lowest-order perturbation theory at large Euclidean momenta. If we carry out a Borel transformation of $\Pi(q^2)$ with respect to the virtuality of the photon q^2

$$\widetilde{\Pi}(M^2) = \lim_{\substack{-q^2,\, n \to \infty \\ -q^2/n = M^2}} \frac{(-1)^n}{(n-1)!} (q^2)^n \frac{d^n}{d(q^2)^n} \Pi(q^2) \tag{1}$$

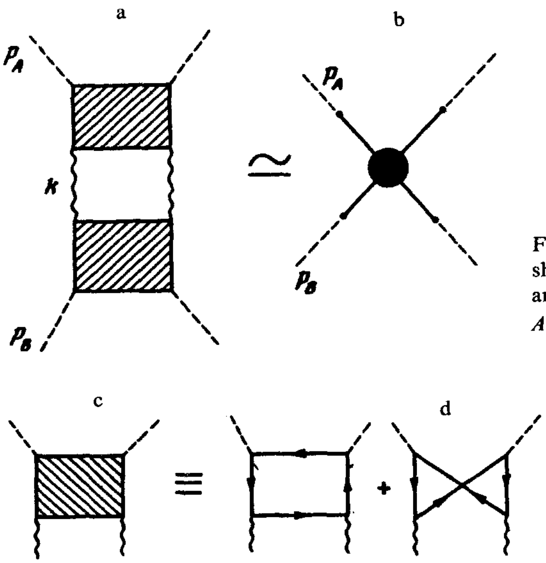

FIG. 1. Schematic representation of the relationship of duality between the photon scattering amplitude $A_{\gamma\gamma}(s)$ and the ρ-meson amplitude $A_{\rho\rho}(s)$.

and take into account phenomenologically the first order corretions to asymptotic freedom by using the parameters $|0|\bar{\psi}\psi|0|, \langle 0|G^q_{\mu\nu}G^q_{\mu\nu}|0\rangle$, then we can obtain an agreement between the theoretical and experimental expressions for $\tilde{\Pi}(M^2)$ in a certain interval of variation of M near the mass m_ρ of the ρ meson.[4] Thus, at the point $M^2 = m^2_\rho$ the first order corrections and the theoretical-perturbative corrections are small ($\approx 10\%$); the contribution from the continuous spectrum to the dispersion integral is of the same order of smallness, and the cross section for e^+e^- annihilation to a ρ meson is almost completely determined by the first diagram of the perturbation theory for $\tilde{\Pi}(M^2)$.

Using the same ideas we shall attempt to calculate the scattering cross section of ρ mesons at high energies. Let us examine the $A_{\gamma\gamma}(s)$ scattering amplitude of two virtual photons with large negative virtualities $q_i^2 = -\lambda_i$ at 0 degrees and attempt to "extract" from it the ρ-meson amplitude $A_{\rho\rho}(s)$. The $A_{\gamma\gamma}(s)$ amplitude is given, on the one hand, by the QCD expression taking into account higher order corrections; on the other hand, it can be written as a dispersion integral over the virtualities of the photons q_i^2 over the intermediate hadronic states. Specifically, this dispersion integral contains a pole contribution corresponding to a transition of the photons to ρ mesons with subsequent scattering of these mesons by each other (Fig. 1b). To effectively isolate the pole term from the contribution of the continuous spectrum and to suppress the higher order in the QCD expression for $A_{\gamma\gamma}(s)$, we carry out a Borel transformation (1) with respect to the virtualities of each photon (we can easily see that the Borel transformation suppresses factorially the remote terms of the power-series and suppresses exponentially the contribution from high energies to the dispersion integral) (see Ref. 4). We can hope that such a "borelized" scattering amplitude of photons at $M^2 = m^2_\rho$ can be illustrated primarily by the first diagram of the perturbation theory (Fig. 1a). It determines the scattering amplitude of ρ mesons at 0 degrees (and hence the total cross section) (Fig. 1b) in terms of the contribution from the ρ-meson poles to the dispersion integral

$$\operatorname{Im} A^{tot}_{\gamma\gamma}(s) = (4\pi\alpha)^2 e^{-2} g_\rho^{-4} m_\rho^{-8} \operatorname{Im} A_{\rho\rho}(s). \qquad (2)$$

After calculating the diagram in Fig. 1a, which is similar in spirit to the calculation of the analogous quantity in quantum electrodynamics,[5] we get

$$\sigma^{tot}_{\rho\rho} = 2\pi\alpha_s^2 \int_0^\infty dk_\perp^2\, \phi^2(k_\perp^2), \qquad (3)$$

where $\alpha_s = g^2(4\pi)^{-1}$ is the strong interaction constant and $k_\perp$ is the perpendicular component of the gluon momentum. The quantity $\phi(k_\perp^2)$ for the cross section averaged over the transverse polarizations of the ρ mesons is given by

$$\phi^2(k_\perp^2) = \int_0^1 dx \int_0^1 dy\, \frac{[1-2x(1-x)][1-2y(1-y)]}{y(1-y)m_\rho^2} \exp - \frac{k_\perp^2 x(1-x)}{m_\rho^2 y(1-y)}. \qquad (4)$$

For longitudinal polarizations we should substitute $8\times(1-x)y(1-y)$ for the expression in the numerator of Eq. (4).

We should like to note the following. Before the Borel transformation the quantity $\phi(k_\perp^2)$, which is proportional to the amplitude of scattering of light by light, depends logarithmically on $k_\perp^2$ in the region of large and small $k_\perp^2$. It is easy to see that Borel transformation eliminates the logarithmic dependence at large $k_\perp^2$, but the logarithm at small $k_\perp^2$ remains. As a result, the region of small momenta in integral (3), which determines the cross section, is given emphasis due to the square of the logarithm $k_\perp^2$ in the integrand. This can be confirmed by a direct calculation: half of the integral over $k_\perp^2$ is collected before the point $k_\perp^2 = 0.25\, m_\rho^2$. If we now substitute in a gauge invariant way perpendicular polarizations for the polarizations in the quark-gluon vertices, then at $k_\perp^2 \ll m_\rho^2$ only the diagram in Fig. 1c will be essential.[6] It follows from this that the impulse approximation, which can be effectively used in a phenomenological description of high-energy hadron-hadron processes, is realized with good accuracy for mesons.[7] The impulse approximation for baryons can conceivably be realized by similar procedure.

To ensure that in the next orders of the pertubation theory the logarithmic corrections $\sim g^2 \ln[k_\perp^2(0.25m_\rho^2)^{-1}]$ for variation of the effective charge are small, we normalize α_s at the integration midpoint $k_\perp^2 = 0.25\, m_\rho^2$. Since the strong interaction constant at this point is 0.5, we can expect that our calculations are relevant with an accuracy to $\sim 3\alpha_s/\pi \sim 0.5$. Substituting $\alpha_s = 0.5$ in Eq. (3) and calculating the integrals, we obtain

$$\sigma^{tot}_{\rho\rho} \approx 15/m_\rho^2 \approx 10 \text{ mb} \qquad (5)$$

for the perpendicular polarizations of ρ mesons, which within our accuracy is in agreement with the 16 mb prediction of the additive quark model. Because of the absence of a logarithmic increase and the numerical smallness of $\phi(k_\perp^2)$, an anomalously small value of 0.2 mb can be analogously obtained for the longitudinal polarizations. It can be shown that the part of the cross section, which depends on the relative orientation of the transverse polarization of ρ mesons, has the same order of smallness.

It is possible to obtain an expression for the scattering cross section of ρ mesons with allowance for the main logarithmic terms in all orders of perturbation theory (see Ref. 2). It appears that when $\alpha_s = 0.5$ the cross section increases too fast compared to the experimental data. Therefore, the expression for the amplitude must be unitarized, a problem that has not been solved yet.

In conclusion, we would like to discuss the problem of the nonperturbative corrections to the $A_{\gamma\gamma}(\alpha)$ amplitude. They can be divided into corrections for the quark–quark interaction (corrections for "interaction") and corrections that must establish, like in the case of vacuum polarization, an agreement between the theoretical and the experimental expressions for the Borel transformation of $\tilde{\Pi}(M^2)$ in a certain interval M near m_ρ (corrections for "stabilization"). The stabilization for each virtuality must be provided for by the power corrections for each of the diagrams in Fig. 1. In QCD, however, the corrections for interaction, because of gauge invariance, are difficult to separate from those for stabilization. This can be done in the case of QED. The expression analogous to (3) is valid for the contribution from the two-photon exchange to the $\rho\rho$ cross section; here the region of momenta $k_\perp^2 \sim 0.25\, m_\rho^2$ is also important. In this region it is natural to use vector dominance for the intermediate photons, which may be considered as a phenomenological allowance for the higher order corrections to the interaction. It appears that the scattering cross section differs negligibly from that calculated according to the pertubation theory. In the case of QED the corrections for stabilization can be calculated in the same way. The authors hope to return to this problem in the next publication.

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