Solitons in nuclear matter

N. N. Kurilkin, I. N. Mishustin, and V. A. Khodel'
I. V. Kurchatov Institute of Atomic Energy

(Submitted 6 August 1979)
Pis'ma Zh. Eksp. Teor. Fiz. **30**, No. 7, 463–467 (5 October 1979)

The propagation of finite-amplitude waves in cold nuclear matter is investigated. It is shown that by taking into account the nonlinear terms in the equations for the Fermi-liquid theory, we obtain elongated wave-soliton-type solutions.

PACS numbers: 21.65. + f

Because of the development of the physics of heavy ions in the last few years, a great deal of interest has arisen concerning nonlinear effects in nuclear systems. The theoretical description of these effects is based essentially on the hydrodynamic approach which has led to the hypothesis of the propagation of shock waves in nuclear matter.[1-3] However, this hypothesis has fundamental problems associated with the fact that nuclear matter is a quantized rather than a classical liquid.

A microscopic treatment of the quantum aspects of the problem in the linear approximation was first attempted by Rumyantsev.[4,5] In this paper we study those new phenomena which are due to nonlinear effects. We shall examine stationary motions of finite amplitude in a cold nuclear matter and show that the propagation of nonlinear waves—solitons is possible. A simple analytical description proposed below is applicable only to waves of relatively small amplitude; however, as we shall show, the resulting physical picture is valid for a more general case.

We examine a stationary, one-dimensional density wave propagating with a constant velocity u in z direction. We denote the variation of the self-consistent nuclear field by $V(\xi)$, where $\xi = z - ut$. The nucleon density $\rho(\xi)$ in this field can be calculated by using the Schrödinger equation. If the characteristic size of the perturbation Δ is large compared to the distance between the particles, it is possible to use the quasi-classical approximation[6,7] which gives

$$\rho(\xi) = \int \frac{k - q_z}{p(\xi)} \left[1 - \frac{mV''}{4p^4(\xi)} - \frac{5(mV')^2}{8p^6(\xi)} \right] \theta(p_F^2 - q^2) \frac{4\, d^3q}{(2\pi)^3} \quad , \tag{1}$$

where $p(\xi)=[(k-q_z)^2 -2mV(\xi)]^{1/2}$ and $k=mu$, where m is the nucleon mass. If $V(\xi)$ has a barrier nature, then at $V(\xi)>(k-p_F)^2/2m$ a classically forbidden region occurs. Thus, only the states with the momentum $q_z\leqslant k-\sqrt{2mV(\xi)}$ give a classical contribution to the density. Calculation of the quantum contribution of the subbarrier region and of the reflected particles requires the use of numerical methods.

The second relationship between ρ and V follows from the condition of self-consistency, which couples them via the local quasiparticle-interaction amplitude $F(\mathbf{r},\mathbf{r}')$. In the rest system of the soliton, where ρ and V are independent of the time, it can be obtained by using the generalized Ward identity for systems with a broken translational symmetry[8] and has the form

$$\nabla V(\vec{\xi}) = \int F(\vec{\xi},\vec{\xi}')\,\nabla\rho(\vec{\xi}')\,d^3\xi'. \tag{2}$$

For F we shall use an expression which in every case is valid for long-wave perturbations:

$$\frac{mp_F}{\pi^2}F(\mathbf{r},\mathbf{r}') = (f_0 + b_0\chi + d_0\nabla^2)\,\delta(\mathbf{r}-\mathbf{r}'), \tag{3}$$

where $\chi(\xi)\equiv\rho(\xi)/\rho_0-1$ and $\rho_0=2p_F^3/3\pi^2$ is the unperturbed density of the nuclear matter. The last term in the square brackets accounts for the dispersion of collective excitations. Unfortunately, the parameters f_0 and b_0 are not known very precisely ($f_0\lesssim 0.5$; $b_0\approx 2$[9]), and there is no experimental data for d_0.

Substituting Eq. (3) into Eq. (2), we obtain

$$\frac{V}{\epsilon_F}\equiv v=\frac{4}{3}\left[f_0\chi+\frac{b_0}{2}\chi^2+d_0\frac{d^2\chi}{d\xi^2}\right]. \tag{4}$$

In summary, we obtain two coupled differential equations for χ and v. From Eqs. (2) and (4) we can obtain the first integral, which reduces the order of the equations:

$$\frac{2d_0}{3}(\chi')^2-d_1(v)(v')^2+W(\chi,v)=0, \tag{5}$$

where

$$d_1(v)=\frac{p_F^2}{4\rho_0}\int\frac{k-q_z}{p^5(\xi)}\,\theta(p_F^2-q^2)\frac{d^3q}{(2\pi)^3}.$$

The "potential energy" $W(\chi,v)$ is determined by

$$W(\chi,v)=\frac{2}{3}f_0\chi^2+\frac{2}{9}b_0\chi^3-\chi v-v+\frac{8}{p_F^2\rho_0}\int(k-q_z)\,[k-q_z-p(\xi)]\times\theta(p_F^2-q^2)\frac{d^3q}{(2\pi)^3}. \tag{6}$$

As the analysis shows, Eqs. (2) and (4) have solitary-wave (soliton) solutions. For small $v<(s-1)^2$ (where $s=k/p_F$). This system is greatly simplified and reduces to a single equation of the Korteveg-de Vries type:

$$D^+(s)\frac{d^2\chi}{d\xi^2}+L^+(s)\chi-\Lambda^+(s)\chi^2=0,$$

$$L^+(s)=1+f^+\Phi(s),\quad \Phi(s)=1-\frac{s}{2}\ln\frac{s+1}{s-1},\quad f^+=2f_0, \tag{7}$$

where

$$D^+(s)=2d_0\Phi(s)-[12p_F^2(s^2-1)^2\Phi(s)]^{-1},$$

$$\Lambda^+(s)=-b_0\Phi(s)+[6(s^2-1)^2\Phi^2(s)]^{-1}. \tag{8}$$

This equation has a well-known solution

$$\chi(\xi)=\chi_0\,\mathrm{sech}^2\frac{\xi}{\Delta}, \tag{9}$$

where

$$\chi_0=3L^+(s)/2\Lambda^+(s),\quad \Delta^2=-4D^+(s)/L^+(s).$$

As $\chi\to 0$, Eq. (7) changes to a known equation in the Fermi-liquid theory for the velocity c_0 of zero sound.[10] Strictly speaking, the solution (6) is valid when s is close to $s_0=c_0/v_F$, and the soliton has a small amplitude $\chi_0\sim(s-s_0)$ and large size $\Delta\sim\left(\frac{1}{p_F}\right)|s-s_0|^{-1/2}$. Thus, it turns out that for $D^+(s_0)>0$ (positive dispersion) $s<s_0$ and the soliton corresponds to a vacuum ($\chi_0<0$), and for $D^+(s_0)<0$ a supersonic ($s>s_0$) condensation soliton is produced ($\chi_0>0$). It is known that the solution of Eq. (7) with $\chi_0>0$ is absolutely stable, whereas for $\chi_0<0$ it is unstable relative to the transverse perturbations.

Other types of solitons may also exist in nuclear matter[11]: spin, isospin, and spin-isospin solitons. The last type is particularly interesting, since the spin-isospin motion is related to the pion degrees of freedom. A single-pion exchange in an effective interaction provides a larger negative dispersion which is necessary to compensate for the nonlinear terms. A spin-isospin wave of finite amplitude $\phi(\xi)$ always induces a change of the scalar density $\chi(\xi)$. Taking this fact into account as was done above, we can obtain a nonlinear Klein-Gordon equation for $\phi(\xi)$:

$$D^-(s)\frac{d^2\phi}{d\xi^2}+L^-(s)\phi-\Lambda^-(s)\phi^3=0, \tag{10}$$

where $L^-(s)=1+g^-\Phi(s)$ and $g^-\approx 1.6$ is a dimensionless amplitude for the interaction of quasi particles in the spin-isospin channel. $D^-(s)<0$ is given by Eq. (8), where $d_0=3m\rho_0f^2/m_\pi^4p_F^2$ ($f=1.0$ is the coupling constant of the πN interaction).

$\Lambda^-(s)$ is a positive function of s whose equation is not given here because of its awkwardness Equation (10) for u larger than the velocity of the spin-isospin sound has a soliton solution:

$$\phi(\xi) = \phi_0 \operatorname{sech}\frac{\xi}{\Delta}, \quad \phi_0 = \frac{2L^-(s)}{\Lambda^-(s)}, \quad \Delta^2 = -\frac{D^-(s)}{L^-(s)}.$$

The developed theory can be used to describe the finite-amplitude waves in other Fermi systems: neutron stars and liquid He^3. We think that nuclear solitons can be produced in collisions of heavy ions of energy ~ 100 MeV per nucleon as a result of decay of the perturbation. The solitons should also accompany a high-energy light ion that passes through a heavy nucleus and forms a nonlinear Mach cone. The spin-isospin solitons should establish a correlation between the spin and the isospin of secondary nucleons and their momentum.

The developed theory can be generalized to include dissipative processes which play a key role in the dynamics of nuclear collisions. All the problems discussed here will be investigated in subsequent publications.

In conclusion, the authors would like to express their deep gratitude to A. B. Migdal, S. T. Belyaev, V. M. Galitskiĭ, Yu. B. Ivanov, V. I Petviashvili, and A. P. Platonov for useful discussions.

[1]G. F. Chapline, M. H. Johnson, E. Letter, and M. S. Weiss, Phys. Rev. **D8**, 4302 (1973).
[2]H. G. Baumgardt, J. V. Schott, Y. Sakamoto, E. S. Schopper, H. Stöcker, J. Hofmann, W. Scheid, and W. Greiner, Z. Phys. **A273**, 359 (1975).
[3]M. Sobel, P. Siemens, J. Bondorf, and H. Bethe, Nucl. Phys. **A251**, 502 (1975).
[4]B. A. Rumyantsev, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 114 (1975) [JETP Lett. **22**, 51 (1975)].
[5]B. A. Rumyantsev, Materialy XII Zinneĭ shkoly LIYaF (Proceedings of the XII Winter School LIYaF) (1977), p. 97.
[6]A. I. Boz', Ya. B. Zel'dovich, and A. M. Perelomov, Rasseyanie, reaktsii i raspody v nerelyativistskoĭ kvantovoĭ mekhanike (Scattering, Reactions, and Decays in Nonrelativistic Quantum Mechanics), Nauka, Moscow (1973).
[7]L. Sham and W. Kohn, Phys. Rev. **137A**, 1697 (1964).
[8]S. A. Fayans and V. A. Khodel', Pis'ma Zh. Eksp. Teor. Fiz. **17**, 633 (1973) [JETP Lett. **17**, 444 (1973)].
[9]A. B. Migdal, Teoriya konechykeh Fermi-sistem i svoĭstva atommykh yader (theory of Finite Fermi-Systems and Properties of Atomic Nuclei), Nauka, Moscow (1965).
[10]L. D. Landau, Zh. Eksp. Teor. Fiz. **35**, 97 (1959) [Sov. Phys. JETP **8**, 70 (1959)].
[11]N. N. Kurilkin, I. N. Mishustin, and V. A. Khodel', Tezisy dokladov XXIX soveshchaniya po yadernoĭ spektroskopii i strukture atomnogo yadra (Abstacts of Reports XXIX Meeting on Nuclear Spectroscopy and the Structure of the Atomic Nucleus), Riga, 37-30 March 1979; Nauka, Leningrad (1979), p. 585.