Gauge theories and variability of the gravitational constant in the early universe

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It is shown that allowance for interaction of the gravitational field with matter leads to a strong modification of the gravitational constant in the early universe, up to a change of its sign.

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1. Recently, the modifications of the general theory of relativity (GTR), in which the gravitational constant G is assumed to vary slowly with time (see, e.g., Ref. 1) have been discussed frequently. We shall show below that the gravitational constant becomes a time-dependent quantity in the standard GTR system, with allowance for the effects associated with interaction of the gravitational field with matter. The modification of the gravitational constant in the early universe is so strong that it can lead to a change of its sign in some cases.

2. It is known that to take into account the interaction of the scalar, vector, and spinor particles ϕ, A_μ, and ψ, with the gravitation, the Lagrangian $L(\phi, A_\mu, \psi)$, in which the simple derivatives are replaced by the covariant derivatives, must be multiplied by $\sqrt{g}$, where $g = \det g_{\mu\nu}$, and we must add to it the Einstein Lagrangian $\frac{\sqrt{g}}{16\pi G} R$, where R is the curvature scalar. To improve certain properties of the theory (conformal invariance, decrease in the number of divergences), the term $\frac{\sqrt{g}}{12} R\phi^2$ is normally added.[2] The total Lagrangian in this case is

$$L = \sqrt{g}\left\{-\frac{1}{16\pi G} R + \frac{1}{12} R\phi^2 + L(\phi, A_\mu, \psi)\right\}. \tag{1}$$

Variation of the Lagrangian (1) with respect to the $g_{\mu\nu}$ metric gives the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}, \tag{2}$$

where $R_{\mu\nu}$ is the Ricci tensor and $T_{\mu\nu}$ is the so-called "improved" energy-momentum tensor obtained as a result of varying the last two terms in Eq. (1).

We note that in the presence of a constant classical field ϕ, the effective gravitational constant, which determines the force of the gravitational interaction of the elementary particles, does not coincide with the G constant in Eq. (2), but, as follows

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from Eq. (1), it is equal to G_ϕ, where

$$\frac{1}{G_\phi} = \frac{1}{G} - \frac{4\pi}{3}\phi^2. \tag{3}$$

In the interaction of particles in the vacuum, the G modification (3) is of little interest and is simply a renormalization of the gravitational constant. However, when the effects associated with the existence of matter in the universe are taken into account, the situation changes dramatically. As is known, in the gauge theories used currently to describe weak, strong, and electromagnetic interactions, the value of ϕ depends on the temperature and density of the medium.[3] For this reason, the effective gravitational constant G_ϕ is also a function of temperature and density, which results in a strong dependence of G_ϕ on the time in the early stages of evolution of the universe.

3. Let us assume, for example, that the density of neutrinos in the universe greatly exceeds the photon density (this does not contradict the current cosmological data[4]). Let us also assume that the coupling constant λ of the scalar mesons ϕ is sufficiently small: $\lambda \ll e^2 \ll 1$, where e is the electron charge (all the qualitative conclusions also hold for $\lambda \sim e^2$). In this case, it follows from Refs. 5 and 6 within the framework of the Weinberg-Salam theory of weak and electromagnetic interactions[7] that the magnitude of ϕ in the early universe (with accuracy to within insignificant corrections associated with the finiteness of the radius of curvature of the universe[8]) was

$$\phi = \left(\frac{e\, n_\nu}{\lambda \sin\theta}\right)^{1/3}, \tag{4}$$

where θ is the Weinberg angle[7] ($\sin\theta \sim 1/2$) and n_ν is the neutrino density. The energy-momentun tensor, with accuracy to within higher corrections in λ and e^2, reduces to the energy-momentum tensor of the ultra relativistic Fermi gas with the equation of state $p = \epsilon/3$, where p is the pressure, and the energy density, ϵ, according to Ref. 6, is

$$\epsilon = T_{00} = \frac{3}{8\pi^2} n_\nu^{4/3}. \tag{5}$$

The evolution of the universe filled with matter with $p = \epsilon/3$ proceeds in a standard manner, see Ref. 9. However, the gravitational constant G_ϕ, according to Eqs. (3)–(5), depends on the density of matter:

$$G_\phi^{-1} = G^{-1} - \frac{4\pi}{3}\left(\frac{e n_\nu}{\lambda \sin\theta}\right)^{2/3} = G^{-1} - \frac{16\pi^2}{3\sqrt{6}}\left(\frac{e}{\lambda \sin\theta}\right)^{2/3}\epsilon^{1/2}. \tag{6}$$

Hence, we can see that for $\epsilon = \epsilon_c$, where

$$\epsilon_c = \frac{54}{(16\pi^2)^2}\left(\frac{\lambda \sin\theta}{e}\right)^{4/3} G^{-2}, \tag{7}$$

the effective gravitational constant is infinite and, as ϵ continues to increase, G_ϕ

changes its sign and asymptotically becomes

$$G_\phi = -\frac{3}{4\pi}\left(\frac{\lambda \sin\theta}{e n_\nu}\right)^{2/3} = -\frac{3\sqrt{6}}{16\pi^2}\left(\frac{\lambda \sin\theta}{e}\right)^{2/3}\epsilon^{-1/2}\ , \tag{8}$$

and decreases to zero as $\epsilon \to \infty$, i.e., near the cosmological singularity.

It should be noted that the critical density ϵ_c at $\lambda \lesssim e^2$ is much smaller than the Planck density $\epsilon_{Pl} \sim G^{-2}$ at which, as is normally accepted, it becomes necessary to take into account the quantum-gravitational effects. Therefore, everywhere at $\epsilon \lesssim \epsilon_c$, except in the small region of the point ϵ_c (as is usually the case in the theory of phase transitions) and also further at $\epsilon_c < \epsilon < \epsilon_{Pl}$, our results are fully reliable. In addition, it can be shown that a decrease of the effective coupling constant G_ϕ near the singularity tends to shift the Planck's density, which increases greatly and is equal to $\epsilon_{Pl} \sim (e/\lambda \sin\theta)^{4/3}\, G^{-2} \gg G^{-2}$.

Thus, a large modification of the gravitational constant in the early universe is not an artifact of the approximation, and if the large quantum-gravitational corrections, near $\epsilon = \epsilon_c$ fail to eliminate the singularity, then at greater densities G_ϕ should be negative, which would correspond to gravitational repulsion. The obtained results may prove to be important in the study of the physical processes occurring in the earliest stages of evolution of the universe.

We note that when the photon density in the universe is greater than the neutrino density (this is the most probable situation), the main contribution to the time dependence of the effective gravitational constant comes from the temperature corrections. The theory of the time variation of the gravitational constant, which is more complex for this case than for that examined above, will be discussed in a separate publication.

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