Damping of one-electron excitations in metals

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It is shown that the scattering of conduction electrons by static defects in metals, although it in itself does not lead to energy relaxation of the excitations, does have a significant influence on the damping of these excitations due to the interaction between electrons. For a sufficiently low excitation energy its damping is proportional to the energy to the three-halves power, and not the square of the energy, as in the Landau theory of a Fermi fluid, which ignores the finiteness of the electron free path.

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The Landau theory of a Fermi fluid, used to describe the conduction electrons in metals, is based on the self-consistent assumption that one-particle excitations behave like free ones, i.e., they weakly interact with one another. In this situation their damping is proportional to ϵ^2—the square of the energy, measured from the Fermi level.[1] This statement does not depend on the specific details of the interelectron interaction and is due to the fact that in scattering of quasiparticles by one another momentum transfer takes place which is substantial and, importantly, independent of the energy. Therefore the damping is determined only by the phase volume, which is proportional to ϵ^2.

In this paper it is shown that when the finiteness of the electron free path l, due to scattering by static defects quite close to the Fermi level, is taken into account, the damping of the quasiparticles is proportional to $\epsilon^{3/2}$, i.e., it falls off more slowly with

energy than in the Fermi fluid theory that ignores the finiteness of the free transit time.

This is the case because the region of small momentum transfers $q \sim \sqrt{\epsilon/D}$ for $\epsilon \ll \tau^{-1}$ ($D = v_F^2\tau/3$ is the quasiparticle diffusion coefficient, v_F is the Fermi velocity) becomes important in addition to the region of large, compared with l^{-1}, momentum transfers (here $\hbar = 1$), which give, as before, a contribution to the damping that is quadratic in ϵ.

Let us note that our result are valid as long as metallic conduction exists, i.e., $p_F l \gg 1$, where p_F is the Fermi momentum.

For simplicity let us first consider the case when the interaction between electrons can be taken into account via perturbation theory. This case is realized if the reciprocal of the Debye shielding radius κ is much less than the Fermi momentum p_F. In this case the energy relaxation of the electrons can be described within the framework of the kinetic equation, which, as shown in Ref. 2, has the form

$$\frac{\partial n_\epsilon}{\partial t} = -\frac{4\pi}{\nu(\epsilon)} \int \frac{d\epsilon' d\omega d^3q}{(2\pi)^5} \left| \frac{4\pi e^2}{q^2 \epsilon(\omega, q)} \right|^2 \mathrm{Re}\, \frac{\zeta_\epsilon \tau \nu(\epsilon)}{1 - \zeta_\epsilon} \mathrm{Re}\, \frac{\zeta_{\epsilon'} \tau \nu(\epsilon')}{1 - \zeta_{\epsilon'}}$$

$$\times [n_\epsilon n_{\epsilon'-\omega}(1 - n_{\epsilon-\omega})(1 - n_{\epsilon'}) - n_{\epsilon-\omega} n_{\epsilon'}(1 - n_\epsilon)(1 - n_{\epsilon'-\omega})]. \tag{1}$$

Here n_ϵ is the number of electrons with energy ϵ, $\nu(\epsilon)$ is the density of electron states, $\epsilon(\omega,q)$ is the dielectric constant of the electron system. The quantity $\zeta_\epsilon(q,\omega)$ is equal to

$$\zeta_\epsilon(q, \omega) = \frac{i}{2ql} \ln \frac{\omega\tau + i + ql}{\omega\tau + i - ql}. \tag{2}$$

In the limit of small energy and momentum transfers ($\omega\tau \ll 1, ql \ll 1$):

$$\mathrm{Re}\, \frac{\zeta_\epsilon(q, \omega)}{1 - \zeta_\epsilon(q, \omega)} = \mathrm{Re}\, \frac{1}{(-i\omega + Dq^2)\tau} = \frac{Dq^2}{(\omega^2 + D^2q^4)\tau}. \tag{3}$$

In this limiting case the dielectric constant has the form

$$\epsilon(\omega, q) = 1 + \frac{D\kappa^2}{-i\omega + D\kappa^2}. \tag{4}$$

For large momentum transfers

$$(q >> l^{-1},\ q >> \omega v_F)$$

$$\mathrm{Re}\, \frac{\zeta_\epsilon}{1 - \zeta_\epsilon} = \frac{\pi}{2ql}, \quad \epsilon(\omega, q) = 1 + \frac{\kappa^2}{q^2}. \tag{5}$$

If the temperature is equal to zero, i.e., $n_{\epsilon'} = \theta(-\epsilon')$, then the energy relaxation time has the form

$$\tau_{ee}^{-1}(\epsilon) = \int_0^{\epsilon} d\omega \int_{-\omega}^{0} \frac{d\epsilon'}{\pi} \int \frac{d^3q}{(2\pi)^3} \left| \frac{4\pi e^2}{q^2 \epsilon(\omega, q)} \right|^2 \tau^2 \nu(\epsilon') \mathrm{Re} \frac{\zeta_\epsilon}{1-\zeta_\epsilon} \mathrm{Re} \frac{\zeta_{\epsilon'}}{1-\zeta_{\epsilon'}}. \tag{7}$$

Let us first consider the contribution to (7) from the region of large q. Substituting (5) into (7), we obtain the well-known result[1]:

$$\frac{1}{\tau_{ee}^{(1)}(\epsilon)} = \frac{\pi^2}{64} \frac{\kappa}{p_F} \frac{\epsilon^2}{\mu}, \tag{8}$$

where μ is the Fermi energy.

The contribution to the relaxation time from the region $\omega \sim \epsilon \ll \tau^{-1}$ and $q \ll l^{-1} \ll \kappa$ (the latter inequality leads to the fact that the interaction potential between electrons can be considered point-like in nature) is obtained when (3) and (4) are substituted into (7). As a result

$$\frac{1}{\tau_{ee}^{(2)}(\epsilon)} = \frac{1}{12\sqrt{2}\pi^3 \nu(0)} \left(\frac{\epsilon}{D}\right)^{3/2} = \frac{\sqrt{6}}{8\pi} \frac{\epsilon^{3/2} \tau^{1/2}}{(p_F l)^2}. \tag{9}$$

Let us note that the region $q \sim \sqrt{\omega/D}$ was found to be important in the integration with respect to q in (7); this was also stated above.

It is seen from a comparison of (8) and (9) that the law $\tau_{ee}^{-1}(\epsilon) \sim \epsilon^{3/2}$ is true for

$$\epsilon << \frac{1}{\tau} \frac{p_F}{\kappa} \frac{1}{(p_F l)^2} = \epsilon_1. \tag{10}$$

As seen from (9), the Debye shielding distance has disappeared since $\nu(0) 4\pi e^2/\kappa^2 = 1$. If the gas approximation ($\kappa \ll p_F$) is not satisfied, then consideration of higher orders of the electron-electron interaction perturbation theory only leads to a renormalization of the effective dimensionless scattering amplitude λ, i.e., the expression on the right side of (9) must be multiplied by λ^2. Essentially, the renormalization of the scattering amplitude, which is necessary in the case when the gas approximation is not satisfied, arises from the region of large transferred momentums. With this renormalization taken into account the interaction between electrons for small momentum and energy transfers can be handled in perturbation theory even when the quantities κ and p_F are of the same order of magnitude, a typical case for real metals.

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[1]A.A. Abrikosov, L.P. Gor'kov and I.E. Dzyaloshinskii, Metody kvantovoi teorii polya v statisticheskoi fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz Press, Moscow, 1962.
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