# Phenomenological laws describing invariant proton emission cross sections for relativistic nuclei interacting with heavy nuclei

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This paper discusses the results of analyzing the data of inclusive experiments on the interaction of $He^4$ and $C^{12}$ nuclei, having an energy of 3.6 GeV/nucleon, with lead nuclei. Certain preferred particle emission directions are found: 67° in the $He^4$ case, 55° in the $C^{12}$ case. It is shown that the relativistically invariant emission cross sections of the secondary protons are described by an expression which is the product of two exponential factors.

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In this paper we report the results of an analysis of experimental data on the interaction of relativistic $He^4$ and $C^{12}$ nuclei with lead nuclei, by means of which certain phenomenological laws were discovered.

  

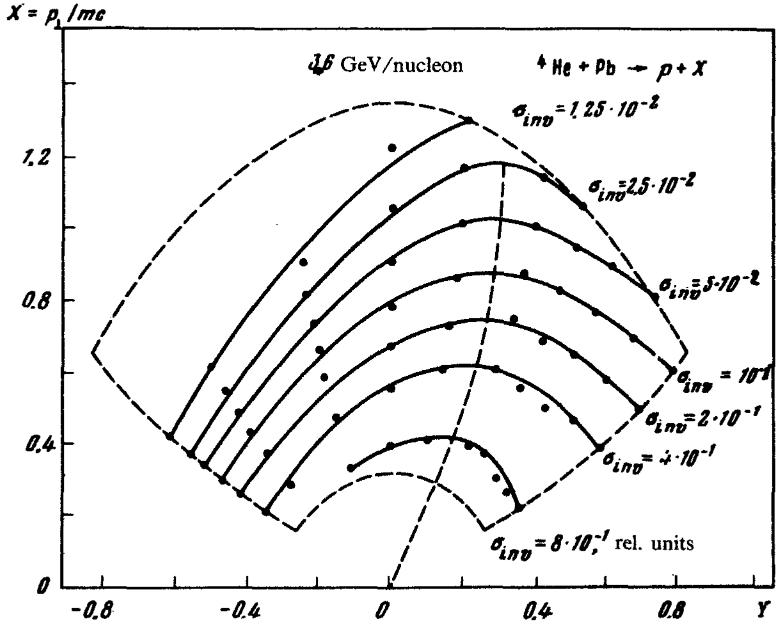


FIG. 1. Different relativistically invariant cross section levels in the $(y,x)$ variable plane.

The experiments were performed on the synchrophasotron at Dubna.[1-3] Extracted beams of $He^4$ and $C^{12}$ nuclei with a energy of 3.6 GeV/nucleon were used. The details of the experiment and the analysis procedure of the primary data have been described in Ref. 3. The relativistically invariant cross sections were found.

The relativistically invariant cross section $\sigma_{\text{inv}}$, was considered to be a function of two variables, the velocity, $y=\frac{1}{2}\ln[(E+p_{\parallel}c)/(E-p_{\parallel}c)]$, and the transverse momentum (in dimensionless units), $x=p_{\perp}/mc$. Here $E$ is the total energy of the particle, $p_{\parallel}=p\cos\theta, p_{\perp}=p\sin\theta$, where $p$ is the momentum and $\theta$ is the emission angle in the laboratory system. A contour chart, depicting the dependence of the invariant proton emission cross section in the $He^4$ + Pb interaction on these variables, is shown in Fig. 1. The dashed lines indicate the boundaries of the domain in the $(y,x)$ plane investigated in the experiment. An examination of this figure reveals one characteristic feature, related to the location of the maximums of the curves corresponding to a constant cross section level. All of these points lie on a line corresponding to one emission angle, namely $\theta_0 \approx 67°$ (dashed line). This unique feature is more clearly evident if the angular dependences $\sigma_{\text{inv}}(x,\theta)$ are plotted for fixed values of the transverse momentum $X$. These curves are shown in Fig. 2. It is seen that the curves have a maximum at an angle of 67° for all values of $X$.

The invariant cross sections in the case of $C^{12}$ + Pb investigated by us also exhibit this same characteristic feature. In this case the maximums of the curves lie on a line corresponding to an emission angle $\theta_0 \approx 55°$.

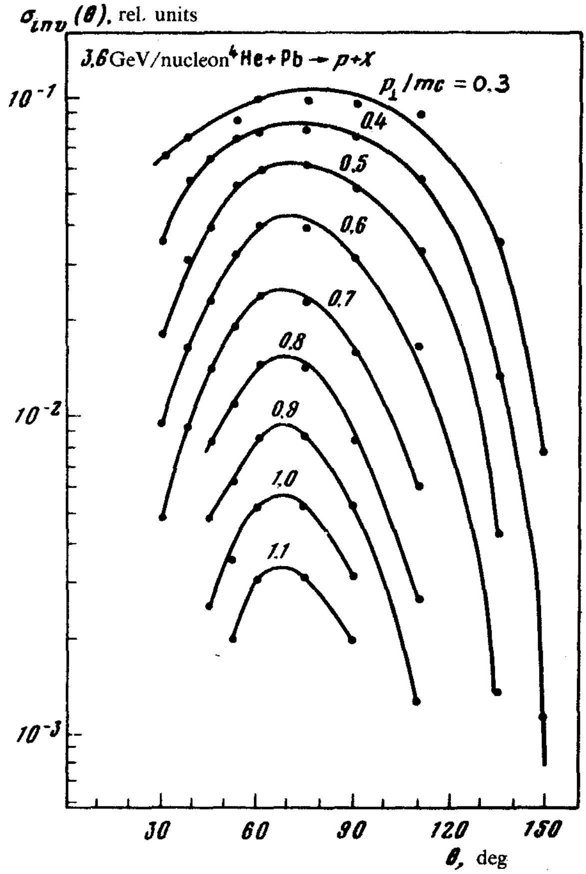


FIG. 2. Dependence of invariant cross section on the observation angle for fixed values of the transverse momentum.

In order to check how general this behavior is, we used the contour charts of the relativistically invariant proton emission cross sections for the interaction of $Ne^{20}$ and $Ar^{40}$ ions, having an energy of 800 meV/nucleon, with Pb, which were published in Refs. 4 and 5. This same behavior can be clearly seen on these charts—the maximum points on the curves of constant $\sigma_{inv}$ lie on lines corresponding to certain fixed values of the emission angle: $\theta_0 \approx 55°$ in the neon case and $\theta_0 \approx 50°$ in the argon case.

That is to say, it can be concluded that a certain preferred nucleon emission direction exists for the interaction of relativistic nuclei with heavy nuclei. The angle of the preferred emission direction apparently depends primarily on the size of the bombarding particle.

To begin with the $\sigma_{inv}^{(max)}(X)$ relationships were analyzed. It was found that in both cases they are well described by the formula

$$\sigma_{inv}^{(max)}(p_\perp) = \sigma_0 \exp\left(-\frac{\sqrt{(p_\perp c)^2 + (m_N c^2)^2} - m_N c^2}{T_\perp^{(0)}}\right), \qquad (1)$$

where $m_N$ is the mass of the nucleon.

A result, similar to this, was previously obtained in Ref. 4 for different cases of nucleus-nucleus interactions at an energy of 800 MeV/nucleon. Let us note here that in both of our cases the values of the constants $T_{\perp}^{(0)}$ were found to be close to each other ($\sim$120 MeV). They are considerably higher than the values found in Ref. 4 ($\sim$70 MeV). A comparison of all the data shows that these constants are apparently determined primarily by the energies of the bombarding particles. Then we examined the dependence of the ratio $\sigma_{\text{inv}}(x,\theta)/\sigma_{\text{inv}}^{(\max)}(X)$ on the difference $\theta - \theta_0$. It is seen from Fig. 2 that the width of the curves increases with a decrease in $X$; therefore this function may be a universal function, not of the change in the angle, but of a combination of the form $X(\theta - \theta_0)$. A natural physical quantity of such a form is $\Delta p_{\parallel}/mc = X(\cot\theta - \cot\theta_0)$—the deviation of the parallel component of the momentum from its value at the angle $\theta_0$. The dependence of $\sigma_{\text{inv}}/\sigma_{\text{inv}}^{(\max)}$ on $\Delta p_{\parallel}/mc$ for all values of $X$ is shown in Fig. 3a for the case $He^4$ + Pb. It is seen from it that the

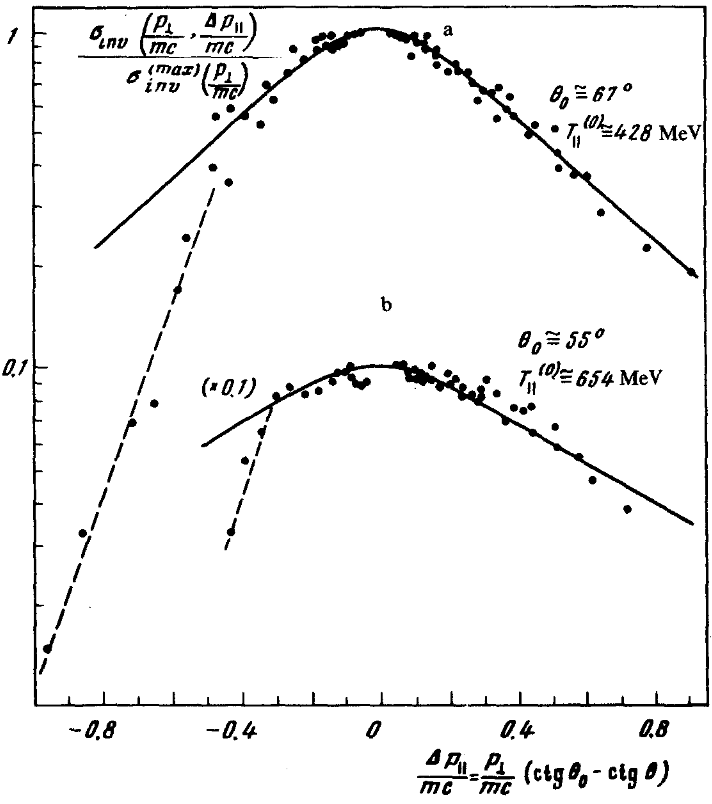


FIG. 3. Dependence of invariant cross section on $\Delta p_{\parallel}$: *a*—for the case $^4He$ + Pb; *b*—for the case $^{12}C$ + Pb. The beam energy is 3.6 GeV/nucleon. Solid curves represent the result of calculation using (2) with the stated parameter values.

experimental points lie on some one curve. It is symmetrical with respect to zero within a wide interval of $\Delta p_{\parallel}$. For $|\Delta p_{\parallel}/mc| \gtrsim 0.2$ the dependence of $\ln[\sigma_{\rm inv}(x,\Delta p_{\parallel})/\sigma_{\rm inv}^{(\rm max)}(X)]$ on $\Delta p_{\parallel}$ is close to a straight line. As $|\Delta p_{\parallel}/mc| \to 0$, it obviously approaches a straight line, parallel to the abscissa axis. At comparatively large negative values of $\Delta p_{\parallel}/mc$ ($\lesssim -0.5$) an abrupt change in its character is observed—the dependence becomes much steeper. In the region $\Delta p_{\parallel}/mc \gtrsim 0.5$ the character of the dependence of $\sigma_{\rm inv}(X,\Delta p_{\parallel})/\sigma_{\rm inv}^{(\rm max)}(X)$ on $\Delta p_{\parallel}$ resembles an exponential, the exponent of which contains the kinetic energy of a particle with momentum $\Delta p_{\parallel}$ and with mass $\lesssim 0.2 m_N$. The natural thing to do was to test the mass of a $\pi$-meson. Therefore we attempted to reproduce the experimental data with a formula of the form

$$\frac{\sigma_{inv}(X, \Delta p_{\parallel})}{\sigma_{inv}^{(max)}(X)} = \exp\left(-\frac{\sqrt{(\Delta p_{\parallel} c)^2 + (m_{\pi}c^2)^2} - m_{\pi}c^2}{T_{\parallel}^{(0)}}\right), \tag{2}$$

where $m_{\pi}$ is the pion mass, and $T_{\parallel}^{(0)}$ is some constant. The solid line in Fig. 3a represents the result of a calculation using (1). It describes the experimental data excellently for all angle and transverse momentum values which correspond to values $\Delta p_{\parallel}/mc \gtrsim -0.5$.

A similar picture (Fig. 3b) is observed in the case of the $C^{12}$ + Pb case. The ratios $\sigma_{\rm inv}(X,\Delta p_{\parallel})/\sigma_{\rm inv}^{(\rm max)}(X)$ depend only on the quantity $\Delta p_{\parallel}$ and this dependence is very well reproduced by Eq. (2) in the region $\Delta p_{\parallel}/mc \gtrsim -0.3$, but with other values of the parameters $T_{\parallel}^{(0)}$ and $\theta_0$. Let us note here that, suprisingly, in both cases (with an accuracy $\sim 10\%$, that to which all constants are determined from the available data) the values of $T_{\parallel}^{(0)}$ are related to the preferred emission angles by the relation $T_{\parallel}^{(0)} = m_N c^2 \cot\theta_0$, and the constants, characterizing the steep fall-off at comparatively large negative $\Delta p_{\parallel}$, coincide with the mass of a $\pi$-meson (dashed line in Fig. 3). Despite the possible variations of the parameter values within the limits of the stated accuracy, it seems reasonable that if the formulas like (2) that are found have a physical meaning, the masses of some real particles should enter into them.

By combining the results of the analysis of the quantities $\sigma_{\rm inv}(X,\Delta p_{\parallel})/\sigma_{\rm inv}^{(\rm max)}(X)$ and $\sigma_{\rm inv}^{(\rm max)}(X)$, it can be concluded that our experimental data are very well reproduced by the following expression:

$$\sigma_{inv}(p, \theta) = \sigma_0 f_1(p_{\perp}) f_2(\Delta p_{\parallel}). \tag{3}$$

In particular, the function $f_1(p_{\perp})$ is determined by the formula (2), and the function $f_2(\Delta p_{\parallel})$ is determined by Eq. (2) for values of the parameters $p$ and $\theta$ corresponding to $\Delta p_{\parallel}/mc \gtrsim -0.5$, and in the region $\Delta p_{\parallel}/mc \lesssim -0.5$ it is steeper, being a quantity that decreases exponentially with an increase in $|\Delta p_{\parallel}|$. Thus, the relativistically invariant cross section can be represented in the form of a product of two factors, indicating the stepwise nature of the process.

In order to clarify the meaning of the phenomenological laws that have been found, extensive theoretical and experimental studies are required. Of considerable interest are studies at much lower energies and, in particular, below the $\pi$-meson production threshold. It is then possible to establish whether the coincidences between the constants and the pion or nucleon masses are accidental, as well as the existence of some preferred direction of collective motion of the nucleons in the nucleus-nucleus interaction process at high energies, which may be due to the formation of a shock wave.

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[1]V.G. Antonenko, V.M. Galitskiĭ, Yu.I. Grigor'yan, M.S. Ippolitov, K.V. Karadzhev, E.A. Kuz'min, V.I. Man'ko, A.S. Ogloblin and G.B. Yan'kov, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 609 (1978) [JETP Lett. **28**, 561 (1978)].
[2]V.G. Antonenko, A.A. Vinogradov, V.M. Galitskiĭ, Yu.I. Grigor'yan, M.S. Ippolitov, K.V. Karadzhev, E.A. Kuz'min, V.I. Man'ko, A.A. Nersesyan, A.A. Ogloblin, V.V. Paramonov and A.A. Tsvetkov, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 103 (1979) [JETP LETT. **29**, 94 (1979)].
[3]V.G. Antonenko, A.A. Vinogradov, V. M. Galitskiĭ, Yu. I. Grigor'yan, M.S. Ippolitov, K.V. Karadzhev, E.A. Kuz'min, A.A. Kurashov, V.I. Man'ko, A.A. Ogloblin, V.V. Paramonov, A.A. Tsvetkov and G.B. Yan'kov, Preprint IAE-3109, Moscow, 1979.
[4]S. Nagamiya, I. Tanihata, S. Schnetzer, L. Anderson, W. Brükner, O. Chamberlain, G. Shapiro and H. Steiner, Preprint LBL-6770, Berkeley, 1977.
[5]S. Nagamiya, L. Anderson, W. Brükner, O. Chamberlain, M.-C. Lemaire, S. Schnetzer, G. Shapiro, H. Steiner and I. Tanihata, Phys. Rev. Lett. B **81**, 147 (1979).