New approach to radiative transitions in charmonium

M. A. Shifman

Institute of Theoretical and Experimental Physics

Radiative decays of the type $J/\psi \to \eta_c \gamma$, $\psi' \to \chi_0 \gamma$ are considered within the framework of the dispersion theory of charmonium. It is shown that for some decays of this type the J/ψ—dominance is parametrically accurate—corrections to it are $\lesssim 0[\alpha_s(m_c)]$ and are negligibly small. The predicted width of $J/\psi \to \eta_c \gamma$ amounts to $\simeq 3.2$ keV if $m_{\eta_c} = 2.977$ GeV.

PACS numbers: 11.20.Fm, 11.30.Jw, 14.40.Pe, 13.40.Hq

At the present time there are two basic theoretical tools for describing $c\bar{c}$ bound states: the nonrelativistic potential model[1-3] and the dispersion theory of charmonium.[4-6] The potential model is very straighforward; unfortunately, however, it permits nearly unrestricted arbitrariness in the choice of the potential. This circumstance drastically reduces its predictive power and yields results of low accuracy and reliability.

The dispersion theory has the necessary rigor, but it does not give a standard formula for calculating all the charmonium characteristics—it must be worked out from scratch in each specific case. Previously lepton widths of the type $\Gamma(J/\psi \to \mu^{+}\mu^{-})$, $\Gamma(\chi_0 \to 2\gamma)$, etc.[4,5] and the mass of the η_c-particle[6] were found within the framework of this theory. It will be shown below that the dispersion approach also makes it possible to obtain a number of relations for radiative decays in charmonium and determines, in particular, the width of $J/\psi \to \eta_c \gamma$ with an accuracy of $\sim 20\%$.

This last decay is of special interest since, according to preliminary SLAC reports,[7] the true η_c-level with a mass of 2.977 GeV has finally been found. The $X(2.83)$—the previous candidate in the role of paracharmonium—has apparently been buried. It is customarily noted that the dispersion theory of charmonium predicted[6] $m_{\eta_c} = 2.98\text{--}3.00$ GeV. If the new prediction is confirmed

$$\Gamma(J/\psi \to \eta_c(2{,}977)\,\gamma) \approx 3.2 \text{ keV},\tag{1}$$

then it is the final assurance that the entire scheme—both in basic aspects as well as in specific details—is correct.

Let us proceed to a systematic presentation. The initial objective is the amplitude of $\eta_c \to 2\gamma$

$$A(\eta_c \to 2\gamma) = \frac{1}{2}\, F^{(1)}_{\mu\nu} F^{(2)}_{\alpha\beta}\, \epsilon_{\mu\nu\alpha\beta}\, f,\tag{2}$$

where $F^{(1,2)}_{\mu\nu} = k^{(1,2)}_{\mu}\epsilon^{(1,2)}_{\nu} - k^{(1,2)}_{\nu}\epsilon^{(1,2)}_{\mu}, k^{(1)}_{\mu}$ and $k^{(2)}_{\mu}$ are four-vector photons, $\epsilon^{(1)}_{\mu}$ and $\epsilon^{(2)}_{\mu}$ are their polarization vector. Moreover, f is an invariant function, depending, generally speaking, on all three ends. Below, however, we will be interested only in the dependence on the one variable $k^{(2)2}$. The relationship between the amplitude f and the probability of the decay $\eta_c \to 2\gamma$ is this:

$$\Gamma(\eta_c \to 2\gamma) = \frac{1}{16\pi} f^2 M^3,\tag{3}$$

where $M \equiv m_{\eta_c}$.

Let us write the dispersion relations in terms of the variable $k^{(2)2}$:

$$f(M^2,\, 0,\, k^{(2)2}) = \frac{1}{\pi}\int\frac{ds}{s - k^{(2)2}}\, \mathrm{Im} f(M^2,\, 0,\, s).\tag{4}$$

The imaginary part, entering into the dispersion integral, breaks down, of course, into a sum of three contributions : J/ψ, the radial excitations (ψ', etc.) and the continuum ($D\bar{D}$, $D\bar{D}\pi\pi$, etc.).

The main point is that only the first of these is large. The radial excitations and the continuum are parametrically suppressed compared with J/ψ; either $\langle v^2\rangle$ (the average velocity of the quarks in η_c) or $\alpha_s(m_c)$ serves as the suppression parameter.

Actually, let us first consider the contribution of the continuum. Its threshold is about $s_{cont \cdot th} \approx 16$ GeV^2, so that $s_{cont \cdot th} - 4m_c^2 \sim 4m_c^2 \gg 1$ GeV^2 (m_c is the mass of a c-quark). As is known, in such a situation the real intermediate states such as $D\bar{D}$, etc., can be replaced with good accuracy by quark-gluon states (so-called hadron-quark duality). If we change to the rest system of the "2"-photon $k^{(2)2} > s_{cont \cdot th}$, the following situation arises. The "heavy" photon creates a pair of "real" quarks $c\bar{c}$, which fly apart with large and oppositely directed momenta $| \mathbf{p} | = \frac{1}{2} (k^{(2)2} - 4m_c^2)^{1/2}$. In order to stick together in η_c, the quarks, after releasing the photon, must be converted into a pair with nearly identical momenta (within the accuracy of the binding energy in η_c). In this case kinematics unequivocally requires that a momentum redistribution occur between the quarks in the photon emission process. In other words, gluon exchange is inevitable. It is easy to prove that the gluon is rigid, the parameter is $(k^{(2)2} - 4m_c^2)^{1/2}$, and the corresponding amplitude contains the small coupling constant $\alpha_s(m_c)$.

Now, what about ψ' and the other excitations in (4). It is obvious that their contribution is equal to zero in the nonrelativistic limit since, in potential language, the overlap integral determining the amplitudes of the type $\psi' \to \eta_c \gamma$ becomes zero. Taking the relativistic corrections $\sim \langle v^2 \rangle \lesssim 0.2$ into account removes the absolute exclusion; however, the smallness of course remains. Thus, if (as assumed in Ref. 3) $\Gamma(J/\psi \to \eta_c \gamma) \sim \Gamma(\psi' \to \eta_c \gamma)$, then the contribution of ψ' to (4) amounts to about 3% of the contribution of J/ψ.

Thus, introducing the amplitudes γ and κ,

$$< 0 | \frac{2}{3} e\bar{c} \gamma_\mu c | J/\psi > = \gamma \epsilon_\mu^{(\psi)} m_\psi^2 ,$$

$$A(J/\psi \to \eta_c \gamma) = \kappa F_{\mu\nu} P_a^{(\psi)} \epsilon_\beta^{(\psi)} \epsilon_{\mu\nu a\beta} , \tag{5}$$

$$(\Gamma(J/\psi \to e^+ e^-) = \frac{\alpha}{3} m_\psi \gamma^2 , \quad \Gamma(J/\psi \to \eta_c \gamma) = \frac{1}{24\pi} \kappa^2 m_\psi^3 (1 - m_{\eta_c}^2 / m_\psi^2)^3),$$

we reduce (4) to the equation

$$f(m_{\eta_c}^2, 0, 0) = \gamma \kappa . \tag{6}$$

within an accuracy of small corrections. In diagram language this result is written in the following manner

$$\Gamma(J/\psi \to \eta_c \gamma) \approx \frac{2}{9} \frac{\Gamma(\eta_c \to 2\gamma)}{\Gamma(J/\psi \to e^+ e^-)} aM(1 - m_{\eta_c}^2 / m_\psi^2)^3 , \tag{7}$$

and means (almost exact) J/ψ-dominance.

The ratio $\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+ e^-)$ was previously found[5,6] in the dispersion theory of charmonium and is numerically close to the nonrelativistic answer[1]

$\approx 4/3$. Using this number, we obtain

$$\Gamma(J/\psi \rightarrow \eta_c \gamma) \approx \frac{64}{27} \frac{a}{M^2} (m_\psi - m_{\eta c})^3, \tag{8}$$

which is identical with the original prediction of the naive nonrelativisitic model, not yet spoiled by recent complications. For $m_{\eta_c} = 2.977$ GeV we obtain (1).

In order to test the reliability of the method, the author calculated the width of $\psi' \rightarrow \chi_0 \gamma$, recently measured experimentally. Let us start with the decay $\psi' \rightarrow h\gamma$ where h is an (auxiliary) mass-less particle with the interaction $h\bar{c}c$. Then the analog of Eq. (7) has the form

$$\Gamma(\psi' \rightarrow \chi_0 \gamma) = g^{-2} \Gamma(\psi' \rightarrow h\gamma) (m_{\psi'}^2 - m_{\chi_0}^2)^3 m_{\psi'}^{-6}, \tag{9}$$

where $g \equiv \langle 0|\bar{c}c|\chi_0 \rangle m_{\chi_0}^{-2}$. Then, according to Wilczek[8] $\Gamma(\psi' \rightarrow h\gamma)$ $= \Gamma(\psi' \rightarrow e^+ e^-)/2\pi\alpha$, and the dispersion theory of charmonium gives[5] $g^2 \approx 1.1 \times 10^{-2}$ (for $m_{\chi_0} = 3.41$ GeV). After substituting the numbers, we obtain

$$\Gamma(\psi' \rightarrow \chi_0 \gamma)_{\text{theor}} \approx 11 \text{ keV}, \tag{10}$$

which must be compared with the experimental value of 16 ± 5 keV.

In conclusion let us note that this paper is genetically related to Ref. 9. Also related is the approach suggested in Ref. 10, although the calculations and, especially, the estimation of the accuracy are a much more complicated problem within the framework of the latter approach.

---

[1] It was assumed that $m_{\eta_c} = 2.98–3.00$ GeV.

---

[1] T. Appelquist and H.D. Politzer, Phys. Rev. Lett. **34**, 43 (1975).
[2] E. Eichten et al., Phys. Rev. Lett. **34**, 369 (1975).
[3] T. Appelquist, R.M. Barnett and K. Lane, "Charm and beyond," in Ann. Rev. Nucl. Particle Sci. **28**, (1978).
[4] V. Novikov et al., Phys. Rev. Lett. **38**, 626 (1977); Phys. Lett. B **67**, 409 (1977).
[5] V. Novikov et al., Phys. Rev. C **41**, (1978); M. Shifman et al. Nucl. Phys. B **147**, 450 (1979).
[6] A. Vainshtein et al., Yad. Fiz. **28**, 465 (1978) [Sov. J. Nucl. Phys. **28**, 237 (1978)].
[7] E.D. Bloom, Talk at the Int. Symp. on Photon and Lepton Interact. at High Energies, Batavia, 1979.
[8] F. Wilczek, Phys. Rev. Lett. **39**, 1304 (1977).
[9] V. Novikov et al., Preprint ITEP-73, 1979.
[10] A. Khodzhamiryan, Preprint ErFI-9, 1979.