# On energetically forbidden supersonic collapse regimes of Langmuir oscillations

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Energy estimates are used to show that a single-mode self-similar supersonic regime of Langmuir oscillation collapse, in which the number of plasmons and the cavity energy are conserved, is impossible for any distribution symmetry of the field and plasma.

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1. One of the central problems of the theory of strong Langmuir turbulence is the investigation of the dynamics of an isolated turbulence cell. The general current position on this question is as follows. Basic equations that describe the dynamics of focusing of the Langmuir oscillations (a system of quasi-hydrodynamic equations that are averaged with respect to "fast" time) were formulated in Ref. 1, and it was shown that under the conditions characteristic of the modulation instability, formation of a singularity was possible. It is natural to expect that the field distribution and concentration near the onset of formation of a singularity are characterized by a self-similar nature. Proceeding from this assumption, self-similar substitutions were proposed in Ref. 1 which reduce the problem to ordinary derivatives that may be used to conclude that the nature of the collapse is supersonic and the number of plasmons in a cavity is conserved. This model in a slightly modified version (since the publication of Refs. 2 and 3 the distribution of fields and plasma in a cavity is considered spherically assymetrical) was used in a majority of works dealing with strong Langmuir turbulence (see the latest review dealing with this question [5]).

Strictly speaking, the conclusion concerning supersonic collapse is based on the results of numerical experiments [4] that cannot be uniquely interpreted for two- and three-dimensional cases. Therefore, analytical results assume a very important role. In this work we shall show on the basis of the integral properties of solutions of the equations of motion that a supersonic self-similar collapse regime is impossible at all field and plasma distribution symmetries.

2. For the sake of brevity and greater convenience we shall confine our analysis to a scalar model of collapse. [6] In this case, a detailed derivation of the equations used below was done in the cited references. The generalization of these for the case of the Langmuir fields fails to evoke difficulties in principal.

We proceed from a supersonic scalar model of collapse. The initial dimensionless system of equations

  

$$
\begin{cases}
i\dfrac{\partial u}{\partial t} + \Delta u - nu = 0, \\[2ex]
\dfrac{\partial \mathbf{v}}{\partial t} = -\nabla |u|^2, \quad \dfrac{\partial n}{\partial t} + \operatorname{div}\mathbf{v} = 0
\end{cases}
\tag{1}
$$

contain a number of integrals of motion. In particular, the number of quanta is conserved

$$
w = \int |u|^2 d\mathbf{r}, \qquad d\mathbf{r} = dx\,dy\,dz \tag{2}
$$

and the energy

$$
H = \int \left( |\nabla u|^2 + n|u|^2 + \frac{\mathbf{v}^2}{2} \right) d\mathbf{r}. \tag{3}
$$

The proposed spherically-symmetric self-similar distribution [6] is similar to the one in Ref. 1 in a manner that it conserves both integrals above. In addition to this, it is understood that such a solution is physically attainable, i.e., it is stable with respect to perturbations. In analogy with known examples from the theory of self-focusing it is natural to assume that the stability calls for the self-similar concentration distribution to be single-mode. [1)] It can be shown that all these requirements (conservation of $H$, $W$, single-modality) are impossible to achieve simultaneously.

Actually, in the vicinity of the onset of formation of a singularity, where the field and concentration distributions are nearly self-similar

$$
(u, n, \mathbf{v}) = (u, n, \mathbf{v})_{\text{self-sim}} + (u, n, \mathbf{v})_{\text{fund}}, \tag{4}
$$

the equations of hydrodynamics for the self-similar velocity and concentration distributions may be readily integrated for any distribution $|u|^2_{\text{self-sim}}$. If

$$
\begin{gathered}
u_{\text{self-sim}} = a^{-3/2}\, u(\vec{\xi}) \exp i \int \lambda^2 dt, \quad \lambda^2 = \lambda_0^2 / a^2, \quad a = (t_0 - t)^{2/3}, \quad \vec{\xi} = \mathbf{r}/a, \\
n_{\text{self-sim}} = a^{-2} N(\vec{\xi}), \qquad \mathbf{v} = a^{-3/2} \nabla \phi(\vec{\xi}),
\end{gathered}
\tag{5}
$$

the first concentration and velocity distributions over the entire space are uniquely defined

$$
\begin{gathered}
\phi(\vec{\xi}) = \frac{3}{2} \xi^{-3/2} \int_0^{\xi} \xi^{-1/2} |u|^2 d\xi, \qquad \phi(0) = |u(0)|^2, \\
N(\vec{\xi}) = \frac{3}{2} \xi^{-2} \int_0^{\xi} \xi\, \Delta \phi\, d\xi.
\end{gathered}
\tag{6}
$$

In the above equation the integration is carried out for fixed angular variables. In the radially-symmetrical case

$$N(\xi) = \frac{3}{2}\, \xi^{-3/2} \frac{d}{d\xi}\, \xi\, (\phi(\xi) - \phi(0)). \tag{7}$$

Distributions of the type of Eqs. (6) and (7) are weakly localized. In accordance with Eqs. (5) and (7), for $\xi \to \infty$

$$N(\xi) = -\frac{3}{2}\, \xi^{2}\, |u(0)|^{2}. \tag{8}$$

The presence of such a weakly-decreasing distribution, as is known from quantum mechanics, leads at a sufficiently large "tail" amplitude to the occurrence in the corresponding potential well of new levels in the discrete spectrum, i.e., to non-single-modality. According to Ref. 7, the necessary condition for the absence of new levels is

$$\frac{3}{2}\, |u(0)|^{2} \leqslant \frac{1}{4} \quad . \tag{9}$$

We shall now calculate the self-similar part of the Hamiltonian. It can be readily seen that $H_{\text{self-sim}} = H_\xi/\alpha^2$, where $H_\xi$ is a functional of the self-similar distribution. Inasmuch as $\alpha(t) \to 0$, clearly, conservation of total energy requires that $H_\xi = 0$. Substituting Eqs. (5), (6), and (7) into (3) leads to an expression

$$H_\xi = \int\left(|\nabla u|^2 - \frac{3}{2}\, |u(0)|^{2} \frac{|u|^2}{\xi^2}\right) d\vec{\xi} + \int\left[\frac{3}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 + \frac{1}{4}\,\frac{\phi^2}{\xi^2}\right] d\vec{\xi}. \tag{10}$$

The only negative term in Eq. (10) is due to $|u(0)|^2 \neq 0$. However, the condition of single-modality [Eq. (9)] leads to the first integral in Eq. (10) being positive on the whole. Actually, as is known (see, for example, Ref. 8)

$$\int |\nabla u|^2\, d\vec{\xi} \geqslant \frac{1}{4} \int \frac{|u|^2}{\xi^2}\, d\vec{\xi}. \tag{11}$$

Therefore, taking Eq. (10) into account we see that a class of single-mode distributions contains

$$H_\xi \geqslant \int\left[\frac{3}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 + \frac{1}{4}\,\frac{\phi^2}{\xi^2}\right] d\vec{\xi} > 0\ . \tag{12}$$

3. All the aforementioned substitutions may also be carried out for the problem of supersonic Langmuir collapse. In this case also the condition of single modality concurrently with the requirement for the constancy of $W$ and $H$ leads to a contradiction. In a special case when $|\vec{\mathcal{E}}(0)|^2 = 0$, we end with the results of Ref. 3 concerning the absence of supersonic self-similar regimes in a spherically-symmetrical case.

Naturally, distributions over limited time intervals may exhibit a quasi-self-similar form which, evidently, is supported by the numerical experiment.[6] However, we imagine that when formulating a theory of the nonlinear stage of the modulation instability in an isolated cell, it is necessary to give up conservation of the plasmon number in a cavity and to remain within the framework of a subsonic collapse regime as is the case for the two-dimensional problem. Moreover, energy dissipation in a cavity is substantially smaller than in the existing calculations.

Finally we should mention that strictly speaking the problem of a scalar collapse is interesting in itself; it is also applicable to electron-phonon coupling in a solid-state plasma, and the problem of strictive self-focusing of three-dimensional clusters of electromagnetic oscillations.

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[1)]Actually, unstable higher self-similar modes are such that the Schrödinger equation with the potential distribution corresponding to these modes contains more than one mode of the discrete spectrum. And, conversely, stable distributions (one-dimensional solution, Townes mode, etc) are single moded.

[1]V.E. Zakharov, Zh. Eksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys. JETP **35**, 908 (1972)].
[2]A.G. Litvak, G.M. Fraĭman, and A.D. Yunakovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 23 (1974) [JETP Lett. **19**, 13 (1974)].
[3]L.M. Degtyarev, V.E. Zakharov, and L.I. Rudakov, Zh. Eksp. Teor. Fiz. **67**, 12 (1974) [Sov. Phys. JETP **40**, 6 (1974)].
[4]L.M. Degtyarev and V.E. Zakharov, Preprint IPM, No. 106 (1974).
[5]L.I. Rudakov and V.N. Tsytovich, Phys. Reports Sect. Conf. Lett. **40**, No. 1 (1978).
[6]O.B. Budneva, V.E. Zakharov, and V.S. Snnakh, Fiz. Plazmy **1**, 606 (1975) [Sov. J. Plasma Phys. **1**, 335 (1975)].
[7]L.D. Landau and E.M. Lifshits, Kvantovaya mekhanika [Quantum Mechanics], M., Nauka, 1974.
[8]O.A. Ladyzhenskaya, Matematicheskie voprosy dinamiki vyazkoĭ neszhimaemoĭ zhidkosti [Mathematical Problems of the Dynamics of Viscous Incompressible Fluid], M., Nauka, 1970.