# Amplifications of far infrared radiation in germanium during the population inversion of "hot" holes

A. A. Andronov, V. A. Kozlov, L. S. Mazov, and V. N. Shastin
*Institute for Applied Physics, USSR Academy of Sciences*

(Submitted 21 August 1979)
Pis'ma Zh. Eksp. Teor. Fiz. **30**, No. 9, 585–589 (5 November 1979)

We show the possibility of enhancing the far infrared (FIR) radiation $\lambda \approx 40$–$400$ $\mu$m due to interband transitions in the valence band of Ge in the case of overpopulation of the light sub-band in crossed electric **E** and magnetic **B** fields at a temperature $T \lesssim 100$ K.

PACS numbers: 78.30.Ev, 71.25.Rk

In materials with degenerate bands, interband transitions (Fig. 1a) make a fundamental contribution to the absorption of FIR radiation.[1] The heating of the holes leads to a change in the absorption spectrum.

Here we wish to draw attention to the fact that in $p$-Ge during hole heating in crossed $\mathbf{E}\perp\mathbf{B}$ fields and inelastic scattering by optical phonons (when the crystal temperature $T \ll \hbar\omega_0$, and the basic scattering process for holes is the spontaneous emission of optical phonons), overpopulation of the $(1 \longleftrightarrow 2)$ transition and the amplification of FIR radiation are possible. Calculations will be given for the absorption (amplification) coefficient of FIR radiation in this case.

At $B = 0$ under these conditions the heating of the holes leads to a strong anisotropy in their distribution[2] and to a decrease[2–5] in the fraction of light holes from $(m_2/m_1)^{3/2}$ to $(m_2/m_1)^2$. A different situation occurs when $B \neq 0$. Here an inversion [6] may occur in the carrier distribution because of the accumulation of carriers in the phase space region near the optical phonon energy, where emission of optical phonons is impossible.[7] These regions occur if the electron drift rate in crossed $\mathbf{E}\times\mathbf{B}$ fields is

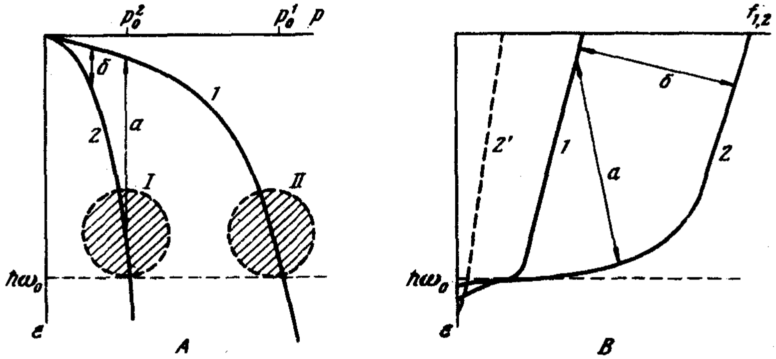

FIG. 1. ($A$) Optical transition in $p$-Ge: $\hbar\omega_0$ is the optical phonon energy; $\epsilon, p$ are the hole energy and momentum; $p_0^\alpha = \sqrt{2m_\alpha\hbar\omega_0}$; $\alpha = 1,2$; $m_{1,2}$ are the hole masses. Possible regions for the accumulation of holes are hatched. ($B$) Qualitative view of the light $f_2$ and heavy $f_1$ hole distributions in strong $\mathbf{E}\perp\mathbf{B}$ fields ($m_1 c E/B \approx p_0^1$); the dashed line is $f_2$ at $B = 0$.

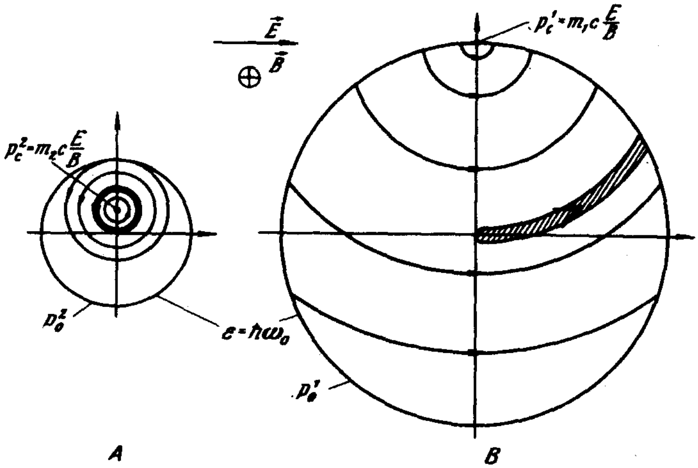

FIG. 2. Trajectories in momentum space for light ($A$) and heavy ($B$) holes at $m_1 c\, E/B \approx p_0^1$. The region of light hole accumulation in strong fields is hatched; also shown are cross sections of the main trajectories, where holes in moderate fields are concentrated.

$v_{\rm dr} = c\,E/B < v_0^\alpha = (2\hbar\omega_0/m_\alpha)^{1/2}$, the rate corresponding to the optical phonon energy [or $p_c^\alpha = m_\alpha\; cE/B < p_0^\alpha = (2m_\alpha\hbar\omega_0)^{1/2}$]. In the combined band (as in $p$-Ge) with increasing magnetic field this region occurs first in the light band (I in Fig. 1a), which gives an overpopulation of the $(1 \longleftrightarrow 2)$ transition, amplification of FIR radiation, and an excess[4,8] in the fraction of light holes above the equilibrium value and even above the fraction of heavy ones.[1] The accumulation of heavy holes in large magnetic fields in a similar region (II in Fig. 1a) should not have a strong effect on the optical properties, since there are no direct optical transitions within the heavy band.

We shall assume the simplest approximation for the Ge valence band, the subbands are isotropic, $m_2 = 0.043$, $m = 0.35$; the phonon scattering frequency is identical for both bands (see, for example, Ref. 5), and we shall neglect impurity scattering. We write the scattering frequency by optical phonons in the form

$$\nu_{opt} = 2\nu_0 \sqrt{\frac{\epsilon - \hbar\omega_0}{\hbar\omega_0}}\,, \qquad \nu_0 = \frac{eE_0}{p_0^1}\,,$$

$$p_0^1 = \sqrt{2m_1\hbar\omega_0}\,(E_0 \approx 4{,}3 \text{ kV/cm}).$$

For the occurrence of an overpopulation of the $(1 \longleftrightarrow 2)$ transition the most favorable situation is when the accumulation region in the light subband is large, and there is none in the heavy one (Fig. 2), while the $E$ and $B$ fields are strong: $E \simeq E_0$, and the cyclotron frequency for the heavy holes is $\omega' = e\,B/m_1 c \approx \nu_0$. In this case, first of all, the accumulation region in the light subband is representative of the entire sphere $\epsilon < \hbar\omega_0$; second, the lifetime of the heavy holes with $\epsilon < \hbar\omega_0 - \tau_1$, which is determined by the time for motion along a trajectory with $\epsilon < \hbar\omega_0$ (see Fig. 2), is small: $\tau_1 \approx 2\,\pi/4\omega c \approx 1/\nu_0$; and third, the heavy holes attain energies $\epsilon \approx \hbar\omega_0$ and, therefore, have practically an equal probability of reaching a state with any energy value $\epsilon < \hbar\omega_0$ after the emission of an optical phonon. Under these conditions it is not difficult to find hole concentrations $N_1$, and $N_2$ in the subbands, and energy distribution functions $f_1 f_2 (N_a \sim \int f_a \sqrt{\epsilon}\, d\epsilon)$. The particle concentrations in the subbands are proportional in this case to the product of the number of states at $\epsilon < \hbar\omega_0$ and the corresponding lifetime (since the probability of a hole reaching a state with $\epsilon < \hbar\omega_0$ is approximately the same here). As long as $\tau_1 \simeq \pi/2\omega_c \approx 1/\nu_0$ in this case, and $\tau_z \approx 1/\nu_a$, where $\nu_a$ is the acoustic scattering frequency, then

$$\frac{N_2}{N_1} \approx \left(\frac{m_2}{m_1}\right)^{3/2} \frac{\nu_0}{\nu_a} \tag{1}$$

the distribution functions $f_1$ and $f_2$ are obtained by a division into phase volumes, i.e.,

$$f_2/f_1 \approx \nu_0/\nu_a\,. \tag{2}$$

For $\epsilon \approx \hbar\omega_0$, $\nu_0/\nu_a \approx 5$ in $p$-Ge at 77 K and $\nu_0/\nu_a \approx 20$ at 5 K. Thus the condition for the overpopulation of the $(1 \longleftrightarrow 2)$ transition, $f_2 > f_1$, may already be satisfied at $T \lesssim 100$ K, while in our case (Fig. 2) it exists for practically all transition frequencies $\omega < \omega_0$.

It is now easy to find the coefficient of absorption of FIR radiation $\mu = \mu_{12} + \mu_{11}$, if we use the well-known equation[1] for the interband coefficient of absorption $\mu_{12} \sim (f_1 - f_2)$, taking into account $f_1$ and $f_2$ from Eq. (2), and the Drude equation for the absorption coefficient in the heavy band

$$\mu_{11} = \frac{2\pi e^2 N_1}{c\sqrt{\epsilon_0}\, m_1 \omega^2}\, \nu_{\text{eff}} \tag{3}$$

where $c\sqrt{\epsilon_0}$ is the speed of light in the crystal, and $e$ is the electron charge. The effective scattering frequency in the heavy band is $\nu_{\text{eff}} \approx 1/\tau_1 \approx \nu_0$. The absorption within the light band is insignificant.

The $\mu$ values calculated in this way are given in Fig. 3. The lower boundary of the gain region $(\mu < 0)$ is determined by the absorption in the heavy holes. The gain remains rather large even at $T = 77$ K, which indicates the possibility of a rather simple use of this effect for generating FIR radiation.

We shall make several remarks in connection with the calculations.

1. The amplification of FIR radiation is also possible for $E \ll E_0$, $\omega_c \ll \nu_0$. Estimates may then be made of the distribution functions and $\mu$, if we take into account the fact that in this case the holes are concentrated near the "main" trajectories (Fig. 2); they indicate that the amplification of FIR radiation occurs at fields up to $E \approx 30$–100 V/cm at 5 K and 500–800 V/cm at 77 K. More precise values may be obtained by numerical modelling (see Refs. 5,6, and 8).

2. The efficiency of energy conversion from a constant electrical field into FIR radiation may reach $(m_2/m_1)^{3/2} \approx 4\%$.

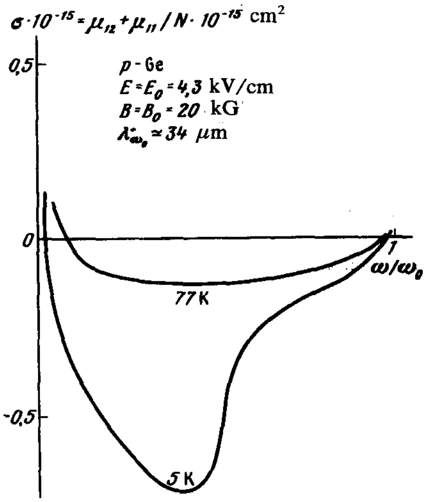

FIG. 3. Absorption cross section $\sigma = \mu/N$ in $p$-Ge in crossed E⊥B fields, $N = N_l + N_2$, and $\lambda_{\omega_0}$ is the wavelength of FIR radiation at $\omega = \omega_0$.

3. The ordinary theory of interband transitions used in the calculations neglects the effect of the **E** and **B** fields on the transition probability. This is justified if the hole energy changes little over the characteristic transition time $\tau \approx 1/\omega$. This leads to the condition $\omega \gg \omega_c^2 = eB/m_2c$. In $p$-Ge at $\omega_c^1 \approx \nu_0$ this means that $\omega \gg 0.16\ \omega_0$.

4. The impurity scattering, which we have neglected, increases the minimum value of the fields at which the amplification of FIR radiation is still possible. In strong fields ($E \approx E_0$, $\omega_c^1 \simeq \nu_0$), however, the gain is preserved even at a relatively high impurity concentration ($N_1 \approx 10^{15} - 10^{16}$ cm$^{-3}$).

In addition to $p$-Ge, other materials ($p$-Si, in particular) with degenerate bands, strong optical-phonon coupling, and a relatively weak acoustic-phonon coupling, are of interest for FIR amplification.

The authors thank V.I. Murzin for discussion and support of the work, V.A. Valov, I.M. Nefedov, and L.Ya. Pavolva for their collaboration, and V.V. Zil'berger for discussions.

[1)] Under these conditions an inversion of the Landau levels may also occur, ODP on TsR[8-11] and static ODP. In $p$-Ge the role of light holes in these effects may also be interesting.

---

[1] A.H. Kahn, Phys. Rev. **97**, 1647 (1955); V.N. Murzin, V.A. Zayats, and V.L. Kononenko, Fiz. T verd. Tela **17**, 2684 (1975) [Sov. Phys. Solid State **17**, 1783 (1975)]; I. Vindevoghel, M. Vindevoghel, and Y. Lerox, Infrared Physics **18**, 99 (1978).

[2] R. Bray and W.E. Pinson, Phys. Rev. **136A** 1449 (1964); L.E. Vorob'ev Yu.K. Pozhela, **12**, 742 (1978) [Sov. Phys. Semicond. **12**, 433 (1978)].

[3] H. Krömer, Proc. IRE **47**, 397 (1959).

[4] I.I. Vosilyus, Fiz. Tverd. Tela **11**, 924 (1969) [Sov. Phys. Solid State **11**, 755 (1969)].

[5] T. Kurosawa and H. Maeda, J. Phys. Soc. Japan **31**, 668 (1971).

[6] H. Maeda and T. Kurosawa, J. Phys. Soc. Japan **33**, 562 (1972); Proc. 11th Inter. Conf. Phys. Semicond., Warsaw (1972), p. 602.

[7] I.I. Vosilyus and I. Levinson, Zh. Eksp. Teor. Fiz. **50**, 1660 (1966). [Sov. Phys. JETP **23**, 1104 (1966)]; *ibid*. **52**, 1013 [*ibid*. **672**, (1967)].

[8] Ya.I. Al'ber, A.A. Andronov, V.A. Kozlov, A.M. Lerner, and I.P. Ryazantseva, Zh. Eksp. Teor. Fiz. **72**, 1030 (1977) [Sov. Phys. JETP **45**, 539 (1977)].

[9] A.A. Andronov and V.A. Kozlov, Pis'ma Zh. Eksp. Teor. Fiz. **17**, 124 (1973) [JETP Lett. **17**, 87 (1973)].

[10] T. Kurosawa, Solid State Physics (in Japanese) **11**, 217 (1976).

[11] T. Kurosawa, Solid State Commun. **24**, 357 (1977).