Self-similar condensation wave in a plasma with a magnetic field

G. E. Vekshteĭn
Institute of Nuclear Physics, Siberian Branch USSR Academy of Sciences

(Submitted 20 August 1979)
Pis'ma Zh. Eksp. Teor. Fiz. **30**, No. 9, 596–600 (5 November 1979)

Heat flow from a magnetized high-pressure plasma is shown capable of being anomalously high, such that the effective thermal conductivity of such a plasma is of the order of Bohm's in the case of classical Coulomb transfer coefficients.

PACS numbers: 52.25.Fi

The pressure of a pulse-heated dense plasma may be considerably higher than the magnetic field pressure ($\beta = 8\pi nT/H^2 \gg 1$) resulting in a contact between the hot plasma and cold walls. The effectiveness of plasma heating in such experiments[1,2] is determined by its cooling rate which is normally estimated from the thermal conductivity of a hot plasma. However, cooling of a plasma with large β is characterized by a number of features associated with the occurrence of plasma flow and the formation of an intermediate boundary layer. As a result of this, a considerable increase in the flow of heat from the hot plasma takes place, and its cooling time becomes much less than the classical.

Let at an initial time a homogeneous hot plasma with the temperature T_0 and density n_0 occupy a half-space $x > 0$, and at $x = 0$ be contiguous to a wall at zero temperature. The magnetic field is parallel to the wall plane and it serves as a suppression of the plasma thermal conductivity. Its value H_0 is such that the plasma is highly magnetized, such that the ion magnetization parameter $(\omega_{Hi}\tau_i)_0 \equiv \delta_0 \gg 1$. Similarly, the magnetic field pressure is low: $\beta_0 = 8\pi n_0 T_0/H_0^2 \gg 1$, and the condition of mechanical equilibrium which is reached during a time much shorter than the plasma cooling time, may be written as follows:

$$nT = \text{const} = n_0 T_0 . \tag{1}$$

The plasma pressure stability above (Eq. (1)) indicates that a decreased temperature in the boundary layer causes an increase in the density, i.e., there occurs a plasma flow to the wall, which must be taken into consideration in the equation of thermal conductivity:

$$\frac{3}{2}\frac{\partial (nT)}{\partial t} = \frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x} + \frac{5}{2}nTv\right) \tag{2}$$

(here and below v is the absolute value of the plasma velocity). From this it follows that the total heat flow in the space remains unchanged and depends on time only:

$$\kappa\frac{\partial T}{\partial x} + \frac{5}{2}nTv = \frac{5}{2}n_0 T_0 v_0 , \tag{3}$$

where $v_0(t)$ is the flow rate of a hot plasma for $x \to +\infty$. As can be seen from the following, a temperature decrease in the boundary layer causes a rapid drop in the plasma velocity v ($v \approx v_0 T/T_0$) such that the convective heat flow is significant only in the hot plasma region with $T \sim T_0$, and at $T \ll T_0$ the principal role is played by the thermal conductivity:

$$\kappa \frac{\partial T}{\partial x} \approx \frac{5}{2} n_o T_o v_o . \qquad (4)$$

Since the coefficient of thermal conductivity κ depends substantially on the magnetic field, Eq. (4) must be solved simultaneously with an equation that describes the evolution of the magnetic field that at $\beta \gg 1$ may be strongly deformed due to the plasma motion and thermoelectrical effects (the Nernst effect)[3]

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left(\frac{c^2}{4\pi\sigma_\perp} \frac{\partial H}{\partial x} + vH + \frac{c}{e} \beta_l \frac{\partial T}{\partial x}\right) . \qquad (5)$$

As can be seen, the Nernst effect leads to additional drifting of the magnetic field from the hot plasma to the wall at a velocity $V_H = (c/eH)\,\beta_L\,(\partial t/\partial x)$, that may be conveniently related to the hot plasma flow velocity v_0. We use a simple model for the transfer coefficients β_l and κ:[3]

$$\beta_l = \begin{cases} (\omega_{He}\tau_e)^{-1}, & \text{at } \omega_{He}\tau_e > 1 \\ \omega_{He}\tau_e, & \text{at } \omega_{He}\tau_e < 1 \end{cases} ; \qquad \kappa = \begin{cases} nc\,T/eH(\omega_{Hi}\tau_i), & \omega_{Hi}\tau_i > 1 \\ nc\,T/eH & 1 < \omega_{He}\tau_e < (M/m)^{1/2} \\ \dfrac{nc\,T}{eH}(\omega_{He}\tau_e), & \omega_{He}\tau_e < 1 \end{cases} \qquad (6)$$

Now, Eqs. (1), (4) and (6) yield

$$V_H = \begin{cases} \frac{5}{2}(m/M)^{1/2} v_o, & \omega_{Hi}\tau_i > 1 \\ \frac{5}{2} v_o/(\omega_{He}\tau_e), & 1 < \omega_{He}\tau_e < (M/m)^{1/2} \\ \frac{5}{2} v_o, & \omega_{He}\tau_e < 1 \end{cases} \qquad (7)$$

It follows from the above that the magnetic field in a hot plasma is forced out by the flow of substance, and in the cold boundary layer the field force-out is associated with the Nernst effect. The magnetic field profile, i.e., solution of Eq. (5), depends substantially on the boundary condition at the wall. Here we shall examine the case of a nonconducting wall, when the effect of increasing thermal losses is particularly strong. Meanwhile the magnetic field at the wall remains equal to its initial value: $H(x=0)=1$. Inasmuch as a characteristic length scale is missing in the problem, the solution should be self-similar, and the self-similar variable should be naturally associated with the hot plasma thermal conductivity χ_0:

$$\xi = x/(\chi_0 t)^{1/2}, \quad T(x,t) = T_0 T(\xi), \quad n = \frac{n_0}{T(\xi)}, \quad H = H_0 H(\xi), \quad v = \left(\frac{\chi_0}{t}\right)^{1/2} u(\xi), \tag{8}$$

where $T(\xi)$, $H(\xi)$, and $u(\xi)$ are dimensionless functions. Increased heat flow at the wall is expressed in these terms such that $u(+\infty) = u_0 \gg 1$. Instead of Eq. (5) we now get:

$$-\frac{\xi}{2}\frac{\partial H}{d\xi} = \frac{d}{d\xi}\left\{\frac{(m/M)^{1/2}}{\beta_0 T^{3/2}}\frac{dH}{d\xi} + (u + V_H)H\right\}. \tag{9}$$

Since at $\xi = 0$, $H = 1$, and $V_H \sim u_0$, the magnetic flow forced on the wall is of the order of the magnetic field flow from the hot plasma, and at $u_0 \gg 1$ these flows are appreciable. This permits us to neglect the left-hand side of Eq. (9), i.e., to consider the magnetic flow to be constant. In addition to this, the contribution of the magnetic viscocity to this flow is small, since the latter contains in the numerator the large parameter β_0 (below the smallness condition will be refined). Thus, the magnetic field is determined from a simple condition:

$$(u + V_H)H = u_0. \tag{10}$$

Considering that $u \approx u_0 T$ and Eq. (7), we get that for $T \gg (m/M)^{1/2}$ the magnetic field is frozen into the plasma, such that $H \approx n = 1/T$. Subsequently, at $T \sim (m/M)^{1/2}$, the Nernst effect enters into play and the magnetic field remains of the order of $(M/m)^{1/2}$ to a temperature $T_1 \sim (m/M)^{1/5}\,\delta^{-2/5}$, where the ions become demagnetized: $(\omega_{Hi}\tau_i)_1 \approx 1$ [we assume $\delta_0 > (M/m)^{3/4}$ and $\beta_0 > M/m$]. Here the magnetic field is discretely reduced to $H_1 = 2/5$ and it remains at this level until $T = 0$ whereupon it becomes unity. Allowance for the magnetic diffusion leads, naturally, to the smoothing of the field profile. It can be shown that at $\beta_0 > M/m$, when the magnetic pressure in the transition layer remains less than the plasma pressure, the discontinuities barely dissolve. Moreover, the magnetic viscocity is less than the plasma thermal conductivity, such that the magnetic field discontinuities are similar to a phenomenon known in hydrodynamics as isothermal density jump in the shock waves,[4] for which the conventional viscocity must be small. The profiles of H, u, *and* V_H are illustrated for clarity.

Knowing the magnetic field, Eq. (4) may be used to determine the temperature profile $T(\xi)$ that contains the heretofore unknown parameter u_0. As a result of this, we find that at $T < T$, $\xi \sim \delta_0^2\,(M/m)^{1/2}\,T^{7/2}/u_0$, the value of ξ in the region $\delta_0^{3/5}\,(m/M)^{1/5}/u_0$ remains virtually unchanged, and, subsequently, ξ increases as it approaches the hot plasma temperature: $\xi \sim (1/u_0)$ arcth $T^{1/2}$ at $T \to 1$. To find u_0, the continuity equation must be used, which may be expressed in the self-similar variables as follows:

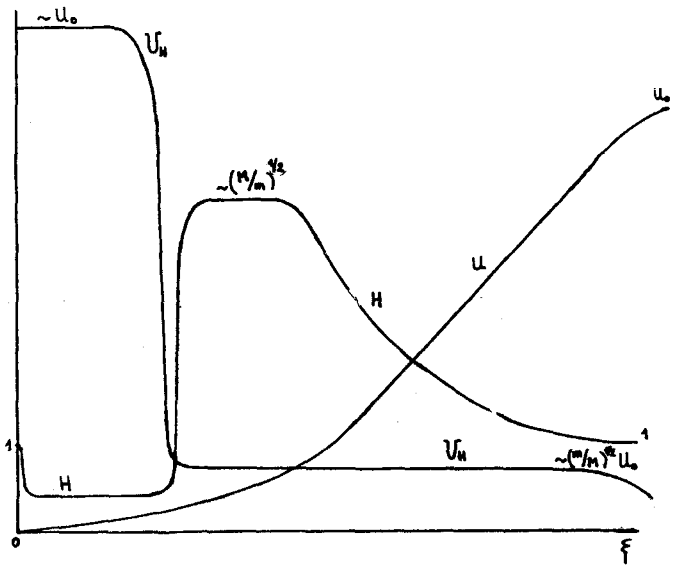

FIG. 1. Structure of the transiton layer in dimensionless variables: H—magnetic field, u—plasma flow velocity, U_H—magnetic field force-out velocity due to the Nernst effect.

$$\frac{\xi}{2}\ \frac{dn}{d\xi}+\frac{d}{d\xi}(nu)=0. \tag{11}$$

Assuming that $n=1/T$, we obtain the plasma flow $nu=\int_0^T[\xi(T)/2T^2]\,dT$ and an equation for $u_0=\int_0^1[\xi(T)/2T^2]\,dT$. The basic contribution to the latter derives from a region with a temperature $T\sim T_1$, and in our model [Eq. (6)] $u_0=(\delta_0/5)^{1/2}\gg 1$. The plasma flow picture is as follows. A stream of matter from the hot plasma "precipitates" in a region $T\sim T_1$, instead of the wall. Therefore, at $T>T_1$ the flow is constant: $nu=\text{const}$ and $u=u_0T$, a fact which we already used earlier. At $T<T_1$ the plasma flow rapidly diminishes and halts at the wall.

If we now return to the normal variables, we find that the heat flux at the wall is $q=(\omega_{Hi}\tau_i)_0^{1/2}n_0T_0(\chi_0/5t)^{1/2}$. Clearly, the effective thermal conductivity of the plasma $\kappa_{\text{eff}}\sim\kappa_0(\omega_{Hi}\tau_i)_0\sim n_0cT_0/eH_0$ is of the same order as the well known Bohm's formula.

[1]E.P. Velikhov, Comments on Plasma Physics **1**, 171 (1972).
[2]R.A. Gross, Nuclear Fusion **15**, 729 (1975).
[3]S.I. Braginskiĭ, Sb. Voprosy teorii plazmy [Coll. Problems of Plasma Theory] **1**, 183 (1963).
[4]L.D. Landau and E.M. Livshits, Mekhanika sploshnykh sred [Mechanics of Continuous Media], M. 1952. p. 362.