A ferroelectric mechanism for color confinement

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A simple mechanism is proposed for color confinement which is based on the concept of a physical vacuum being a physical medium with spontaneous polarization.

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It is well-known that the problem of color confinement is still awaiting a solution (a review with a detailed bibliography is given in Ref. 1). In addition, there is a basis for believing that the definite form for the ordering of the vacuum serves as a physical reason for the confinement. In this situation it makes sense to formulate the inverse problem: to find those types of ordering which lead to a confinement pattern. The solution of this problem would not only lead to a phenomenological theory of confinement, but would also indicate which instability modes should be looked for in the dynamic equations for the microscopic basis of this theory.

In this article it is shown that, in addition to the superconducting (Higgs) type of vacuum ordering that is usually considered, another, even simpler, mechanism leads to confinement. It occurs in the case when the vacuum, similar to a "soft" ferroelectric (pyroelectric), has a spontaneous polarization whose direction is determined by the direction of the external field at a given point in space.

1. A specific reaction of the vacuum to the color charge field, which is related to its properties as an ideal "dielectric," a medium with dielectric constant $\epsilon \to 0$ (the electrical analog of an ideal diamagnetic, a superconductor), forms the physical basis of the most popular "string" mechanism of confinement. From the minimum energy (its density is proportional to D^2/ϵ) condition it follows that as $\epsilon \to 0$ there is a pushing-out of the induction **D** lines of force and, because of the constancy of its flux, their concentration appears in the form of a quasi-one dimensional string. Correspondingly, the linear dependence of the interaction energy on r, $u \to e E_0 r$ (where e is the positive charge of the particle and E_0 is a constant) corresponds to a large distance r between the charges, and this also indicates confinement.[1)]

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Proceeding from this relationship, it is possible to look directly for a physical equation for the dielectric vacuum, i.e., the relationship between the induction and the field intensity for **E**. Determining from U the force acting on a particle and relating it to **E**, we have

$$\mathbf{E} = -E_0 \mathbf{r}/r, \tag{1}$$

after which the problem is reduced to an expression of the right side of Eq. (1) in terms of **D**.

2. We shall assume initially that the physical equation is linear at spatially nonlocal, $\mathbf{D}(\mathbf{k}) = \epsilon(\mathbf{k})\mathbf{E}(\mathbf{k})$. Then, Eq. (1) and the equation

$$\operatorname{div}\mathbf{D} = 4\pi e(\delta(\mathbf{x}) - \delta(\mathbf{x}-\mathbf{r}))$$

$$\epsilon(\mathbf{k}) = ek^2/2E_0, \tag{2}$$

yield an equation for the dielectric constant which vanishes as $k \to 0$ ($r \to \infty$). This corresponds to the superconducting (Meissner) mechanism: the magnetic permeability of superconductor has a similar form $(1 + \mathrm{const}/k^2)^{-1}$.

An alternative is a nonlinear, although local, material equation. As long as the directions of the vectors **D** and **E** coincide in a stable isotropic vacuum,[2] Eq. (1) gives directly

$$\mathbf{E} = E_0 \mathbf{D}/D. \tag{3}$$

This material equation is valid in the limit $\mathbf{D} \to 0$ ($r \to \infty$). It corresponds to the permittivity tensor $\epsilon_{ij} = \partial D_i/\partial E_j$, whose transverse (relative to the direction for **D**) eigenvalues equal D/E_0 and vanish as $D \to 0$. In the same limit, the polarization (dipole moment density) differs from zero

$$\mathbf{P} = (\mathbf{D} - \mathbf{E})/4\pi \to -E_0 \mathbf{D}/4\pi D, \tag{4}$$

i.e., it has a spontaneous character.[2)] The solution of Maxwell's equations along with the material equation (3) actually describes an ideal string

$$\mathbf{D}(\mathbf{x}) = 4\pi e\mathbf{r} \int_0^1 dt\, \delta(\mathbf{x} - t\mathbf{r}). \tag{5}$$

3. The development of the preliminary considerations described above will be the subject of subsequent publications. Here we shall limit ourselves to a few general remarks.

We shall show first that the derivation of an equation of the type (3) from a dynamic theory should be based, as in the theory of ferroelectricity (see Ref. 3), on the concept of free energy (the effective Lagrangian) in the form of an expansion in powers of the induction or polarization and the appearance of an "incorrect" sign for the corresponding coefficients.

We note further that the presence of a uniform electric field (see Ref. 3) does not mean, as might have been thought, instability of the vacuum. In the usual electrodynamics this would actually be so because of pair production and the unrestricted acceleration of their components. However, in the case we are considering, the very fact of confinement prevents the occurrence of this process.

Finally, we emphasize that in an ordinary ferroelectric there are neither strings nor charge confinement. Its dielectric constant is greater than unity (paraelectric). Meanwhile, in an ideal paraelectric ($\epsilon \rightarrow \infty$) the field intensity lines of force (energy density proportional to ϵE^2) should be expelled and form a string, but this contradicts the equation rot $\mathbf{E} = 0$.

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[1)]For simplicity we have limited ourselves to discussion of an Abelian electric field and a "quark-antiquark" system.

[2)]The negative sign in Eq. (4) is related to specific anti-screening phenomena for quark interaction (asymptotic freedom), which corresponds to $\epsilon < 1$.

[1]B. Sakita, Quantum chromodynamics and related problems, Proc. XIX Intern. Conf. on High Energy Physics, Tokyo (1978).

[2]L.D. Landau and E.M. Lifshits, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Moscow (1959).

[3]V.L. Ginzburg, Zh. Eksp. Teor. Fiz. **15**, 739 (1945); F. Iona and D. Shirane, Segnetoelektricheskie kristally (Ferroelectric Crystals), Moscow (1965).