Possibility of combined self-focusing of atomic and light beams

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(Submitted 20 July 1979; resubmitted 12 October 1979)
Pis'ma Zh. Eksp. Teor. Fiz. **30**, No. 10, 645–647 (20 November 1979)

The effect of combined self-focusing and self-channeling of coaxial atomic and light beams is predicted theoretically.

PACS numbers: 42.65.Jx, 41.80.Gg

The first observation of focusing of a beam of Na atoms by a superimposed light beam as a result of light pressure dipole forces was reported in Ref. 1. Our goal is to focus attention on the possibility of also focusing light in such an experiment. When atomic and light beams propagate jointly (along the z axis) the intensity of the light field can be represented as follows:

$$\mathbf{E}(\mathbf{R}, t) = \frac{1}{2} \mathbf{E}_{\omega,\mathbf{k}}(\mathbf{R})\, e^{-i\omega t + ikz} + \text{compl. conj.} \tag{1}$$

The state of the atom-field system, in which the force acting on the atoms is

$$\mathbf{F}(\mathbf{R}, t) = \mathbf{k}\frac{\operatorname{Im}\alpha(\omega,\mathbf{k})}{2n_a}\left|\mathbf{E}_{\omega,\mathbf{k}}\right|^2 + \frac{\operatorname{Re}\alpha(\omega,\mathbf{k})}{4n_a}\operatorname{grad}_R\left|\mathbf{E}_{\omega,\mathbf{k}}\right|^2 . \tag{2}$$

is established during the relaxation of polarization γ^{-1}.[2,3] In deriving Eq. (2) we assumed that $V\gamma^{-1} \ll \delta R$, where δR is the characteristic dimension of the inhomogeneity of the field. In Eq. (2) $\alpha(\omega,\mathbf{k})$ is the susceptibility of the system of two-level atoms:

$$\alpha(\omega, \mathbf{k}) = -\frac{n_a d^2}{\hbar}\frac{D}{\Omega + \mathbf{kV} - i\gamma}, \qquad D = D^\circ \frac{(\Omega + \mathbf{kV})^2 + \gamma^2}{(\Omega + \mathbf{kV})^2 + \gamma^2\left(1 + a\left|\mathbf{E}_{\omega,\mathbf{k}}\right|^2\right)} \tag{3}$$

Here $\Omega = \omega_0 - \omega$, n_α is the volume concentration of atoms, α is the saturation parameter, d is the dipole moment, and D^0 is the difference between the populations in zero field. We assume that detuning does not change substantially in the process of interaction of the atom with the field:

$$k\delta V_z(t) << \Omega + kV_{zo} = \Omega_o \quad \text{when } t << t_1. \tag{4}$$

Under condition (4) we can introduce the potential of the dipole force directed along $\text{grad}_R |\mathbf{E}_{\omega,\mathbf{k}}|$:

$$F_p = -\frac{\partial U(\rho)}{\partial \rho}, \quad U(\rho) = \frac{1}{2}\hbar\Omega_o D^\circ \ln \frac{\Omega_o^2 + \gamma^2(1 + a|\mathbf{E}_{\omega,\mathbf{k}}|^2)}{\Omega_o^2 + \gamma^2}. \tag{5}$$

In the time interval $\gamma^{-1} \ll t \ll t_1$ we can assume that the velocity distribution in the beam cross section is Maxwellian with a temperature $T_{\rho 0}$; thus, the distribution of the atoms $n_\alpha(\rho)$ along the radius is determined by the Boltzmann distribution:

$$n_a(\rho) = n_{ao} \exp\left(-\frac{U(\rho)}{k_B T_{\rho o}}\right), \tag{6}$$

where $n_{\alpha 0}$ is the vapor concentration in the atomic source. Generally, the temperature T_ρ is not constant along the z axis. The fact is that the force of light pressure fluctuates due to the random nature of photon radiation.[4] This produces stochastic heating of atoms by the field to the transverse energies which are larger than the height of the potential barrier of the dipole force $U(0)$ and eventually destroys self-focusing. We, therefore, assume that $t \ll t_2$ heating time of the atoms.

The dependence of the polarizability α on ρ is determined by two functions $n_a(\rho)$ and $|\mathbf{E}_{\omega,\mathbf{k}}|^2$ as follows from Eq. (3). They also determine the dependence of the refractive index on ρ:

$$n(\rho) \approx 1 + 2\pi \text{Re}\, a = 1 - \frac{2\pi d^2 D^\circ \Omega_o}{\hbar} \frac{n_a(\rho)}{\Omega_o^2 + \gamma^2(1 + a|\mathbf{E}_{\omega,\mathbf{k}}(\rho)|^2)}. \tag{7}$$

It follows from this that $\Omega_0 < 0$ the saturation effect leads to focusing of the light beam and at $\Omega_0 > 0$ it leads to its defocusing.[5] At $\Omega_0 > 0$, however, the concentration of atoms in the region with $|\mathbf{E}_{\omega,\mathbf{k}}|^2 \neq 0$ increases. Thus, the field-dependent $n_E = n - n_0$ part of the refractive index also increases. Substituting Eq. (6) in Eq. (7), we determine the dependence of the refractive index on the intensity of light

$$n_E = -\frac{2\pi d^2 D^\circ n_{ao}}{\hbar\Omega_o}\left\{\left[\exp\left(-\frac{U(\rho)}{k_B T_{\rho o}}\right) - 1\right] - \frac{\gamma^2 a|\mathbf{E}_{\omega,\mathbf{k}}|^2}{\Omega_o^2}\right\}, \tag{8}$$

where it was assumed that $\Omega_0 \gg \gamma$ and $a|\mathbf{E}_{\omega,\mathbf{k}}|^2 \ll \Omega_0^2/\gamma^2$. Substituting in Eq. (8) expression (5) for the potential of the dipole force, we can see that the sign of the effect (self-focusing or self-defocusing) depends on the relation between the energies $\hbar\Omega_0$ and

$2k_B T_{\rho 0}$. The saturation effect in gases always predominates over electrostriction because $2k_B T_{\rho 0} \gg \hbar \Omega_0$ even at the critical point. The reciprocal inequality (i.e., self-focusing of light at $\Omega_0 > 0$) can only be realized in a beam.

Thus, if $\Omega_0 > 0$, $D^0 < 0$ (or conversely, $\Omega_0 < 0$, $D^0 > 0$ for a beam of atoms with an inverse population), then the dipole forces of light pressure may be the cause of combined self-focusing of atoms [the $U(\rho)$ potential is smaller on the axis of the beams] and of light [the refractive index $n(\rho)$ is larger at $\rho = 0$]. We give numerical estimates of the main parameters. Let us assume that the atomic and light beams fill a cylinder of cross section S uniformly. Equating the angle of diffractive divergence of light to the angle of total internal reflection,[6,7] we determine the linear concentration of the atomic beam $p_\alpha = S n_\alpha$ necessary for self-channeling of light:

$$p_a = 0.023 \frac{\lambda^2 \hbar}{d^2 \Omega_0} [\Omega_0^2 + \gamma^2 (1 + a |E_{\omega,k}|^2)]. \tag{9}$$

For the D^2 sodium line ($3^2S_{1/2} - 3^2P_{3/2}$ transition) at light power $W = 600$ mW, which is permissible for dye lasers, $S = 10^{-4}$ cm^2 and $\Omega_0 = 10^{11}$ sec^{-1}, we obtain $p_\alpha = 6.3 \times 10^8$ cm^{-1}, $n_\alpha = 6.3 \times 10^{12}$ cm^{-3}, $t_1 = 3.4 \times 10^{-1}$ sec, and $t_2 = 2.6 \times 10^{-2}$ sec. The height of the potential barrier of the dipole force for these parameters is $U(0) = 1.6 \times 10^{-18}$ erg, which means that the sodium atoms with transverse velocities $V_{\rho 0} < 290$ cm/sec may be captured by light. For $V_{z0} = 7 \times 10^4$ cm/sec this corresponds to the initial angular divergence of the atomic beam $2\theta_0 = 8.3 \times 10^{-3}$. The distances $l_i = v_{z0} t_i$ are large: $l_1 = 2.4 \times 10^4$ cm and $l_2 = 1.8 \times 10^3$ cm, which is close to the attenuation length of the field $l_3 = 6.1 \times 10^3$ cm. Because of hyperfine splitting of the ground state of sodium ($\delta\omega = 1.11 \times 10^{10}$ sec^{-1}) and thermal velocity straggling of the atoms ($\delta\omega_D = 7 \times 10^9$ sec^{-1} at $T = 600$ K) the given values are approximate.

We assumed that the atomic and light beams propagate in one direction. This, however, is not necessary. The atomic and light beams can move in the opposite direction. There are also reasons to assume that simultaneous focusing of atomic and light beams (simultaneous "compaction") is easier to achieve when the distribution of the field and the atoms is "symmetrized." This can be achieved by using colliding atomic-light beams.

[1]J. E. Bjorkholm, R. R. Freeman, A. Ashkin, and D. B. Pearson, Phys. Rev. Lett. **41**, 1361 (1978).
[2]G. A. Askar'yan, Zh. Eksp. Teor. Fiz. **42**, 1567 (1962) [Sov. Phys. JETP **15**, 1088 (1962)].
[3]A. Ashkin, Phys. Rev. Lett. **25**, 1321 (1970).
[4]A.Yu. Pusep, Zh. Eksp. Teor. Fiz. **70**, 851 (1976) [Sov. Phys. JETP **43**, 441 (1976)].
[5]A. Javan and P. L. Kelley, IEEE JQE **2**, 470 (1966).
[6]R. Chiao, E. Garmire, and C. Townes, Phys. Rev. Lett. **13**, 479 (1964).
[7]S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Usp. Fiz. Nauk **93**, 19 (1967) [Sov. Phys. Usp. **10**, 609 (1967)].