Transverse conductivity in quantizing magnetic field for the case of finite-radius scatterers

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(Submitted 12 October 1979)
Pis'ma Zh. Eksp. Teor. Fiz. **30**, No. 10, 665–668 (20 November 1979)

The divergent-free transverse conductivity of an electron gas in a quantizing magnetic field was determined by allowing for small mixing of the Landau levels by scatterers of finite radius.

PACS numbers: 72.10.Di

1. The transverse conductivity is usually calculated in the Born approximation from the interaction of the current carriers with the scatterers. A logarithmic divergence at small energies,[1] which can be eliminated by introducing one or another cutoff method, arises in this case in a quantizing magnetic field. Skobov[2] showed that not using perturbation theory in the case of small-radius scatterers automatically eliminates the divergence.

Below we derive a formula for the transverse conductivity of noninteracting electrons scattered by centers of finite but arbitrary radius α in a quantizing field assuming that: a) mixing of the Landau levels by an individual $U(|\mathbf{r}|)$ center is small. We emphasize that the potential U is not assumed to be small compared to the energy of the longitudinal motion. It is shown that there is no divergence[1] for centers of finite radius. On the basis of the obtained formulas the energy spectrum of the neutral impurities in semiconductors can be analyzed from the dependence of the conductivity on temperature and on the magnetic field.

2. The diagonal component of the conductivity tensor has the form[2]:

$$\sigma_{yy} = \frac{e^2}{2\pi V T} \operatorname{Re}\left[\operatorname{Sp}\hat{\rho}\hat{v}_y \int_{-\infty}^{+\infty} dE \frac{1}{(E-\hat{H}-i\epsilon)} \hat{v}_y \frac{1}{(E-\hat{H}+i\epsilon)}\right] \tag{1}$$

$\hat{H}$ is the complete Hamiltonian of the system in the absence of the electric field, V is the volume, and $\hat{v}_\gamma$ is the electron-velocity operator. In the $(T/\hbar)\tau \gg 1$ approximation the current in the system is the product of the volume averaged current of an individual scatterer and the number of scatterers in the volume. In determining the current from an individual scatterer, the Hamiltonian of the one-center problem $\hat{H}_0$ must be substituted for the $\hat{H}$ Hamiltonian. In the expression obtained from Eq. (1) we take Sp in the complete system of wave functions of the one-center problem. We can take an individual scatter at the origin without loss of generality. Since the $\hat{v}_\gamma$ matrix elements are nonvanishing only for the transition between the Landau bands $N \to N \pm 1$, we construct the wave function of the one-center problem taking into account (a): we solve the Schrödinger equation in the zero band, omitting the other bands, and then add the first band according to perturbation theory. Characterizing the electron state

by the quantities p_z, n, and m,[3] where p_z is the momentum along **H**, and n and m are the radial and aximuthal quantum numbers, respectively, we obtain:

$$\sigma_{yy} = 4\pi^3 e^2 n_0 l^4 \sum_{m=0}^{\infty} (m+1) \int_{-\infty}^{+\infty} dp_z dp \left(-\frac{\partial \rho}{\partial \epsilon_p}\right) \delta(\epsilon_{p_z} - \epsilon_p) \times \left| \int_{-\infty}^{+\infty} dz \, \chi^{p_z^*}_{m+1}(z) [U_m(z) - U_{m+1}(z)] \chi^{p}_{m}(z) \right|^2 \tag{2}$$

$l = (c\hbar/|e|H)^{1/2}$ is the magnetic length, $U_m(z) = \langle R_{0m}(\rho)/U(\rho,z)R_{0m}(\rho)\rangle$, and R are the radial wave functions.[3] χ^p_m is the solution of the one-dimensional problem for the scattering of an electron with momentum p by the $U_m(z)$ potential, and n_0 is the concentration of the scatterers.

3. Substituting in Eq. (2) the plane waves for the χ functions, we obtain the conductivity in the Born approximation and Eq. (2), after certain transformations, is reduced to the Adams and Holstein result.[1] In case of a shallow $U_0 \lesssim \hbar^2/m^*a^2$, short-range $a \ll l$ potential, assuming that $U_{m>1} = 0$ and substituting the plane wave for χ_1, we obtain the Skobov result.[2]

For the potential of an arbitrary finite radius it follows from the boundary conditions that in the general case of small energies ($p \to 0$) the wave function χ is proportional to p in the range of the $U_m(z)$ potential, which contributes to the transverse conductivity (2); i.e., there is no divergence[1] for a potential of finite radius, and this in turn means that in the region of low temperatures all temperature and magnetic-field dependences of the conductivity, which are predicted by the Born approximation theory will change.[1] The last case is of considerable experimental interest in view of the possibility of investigating the energy spectrum of the impurities in a quantizing magnetic field from the dependence of σ on H and T. We shall show this for the case of shallow, $U \lesssim \hbar^2/m^*a^2$ impurities. For such centers the conductivity, which can be calculated according to Eq. (2), is (the electron spectrum is assumed to be quadratic):

$$\sigma_{yy} = \frac{\pi e^2 n_0 \hbar^2 l^2}{m^*} \sum_{m=0}^{\infty} (m+1)(\gamma_m - \gamma_{m+1})^2 \int_0^{\infty} \frac{d\epsilon}{\epsilon} \left(-\frac{\partial \rho}{\partial \epsilon}\right) \frac{1}{\left(1 + \frac{\gamma_m^2}{2\epsilon}\hbar\omega_H\right)} \times \frac{1}{\left(1 + \frac{\gamma_{m+1}^2}{2\epsilon}\hbar\omega_H\right)} \tag{3}$$

$$\gamma_m \sim U\left(\frac{a}{l}\right)^{(2m+1)} \quad \text{when } l \gtrsim a;$$

At $l > a$, $\gamma_m^2 \gg \gamma_{m+1}^2$. In this case, we can limit ourselves in Eq. (3) to the first term with

$m = 0$, but both factors with γ_0 and γ_1 should be retained in the denominator of integrand (3). Assuming that the electron distribution function is a Boltzmann function, we obtain different H and T dependences for different temperature regions

$$\sigma_{yy} \sim T^{-3/2} \ln \frac{T}{H^2}, \qquad T >> \frac{\gamma_0^2}{2} \hbar\omega_H ,$$

$$\sigma_{yy} \sim \frac{T^{-1/2}}{H^2}, \qquad \gamma_0^2/2\, \hbar\omega_H >> T >> \frac{\gamma_1^2}{2} \hbar\omega_H ,$$

$$\sigma_{yy} \sim \frac{T^{1/2}}{H^6}, \qquad T << \frac{\gamma_1^2}{2} \hbar\omega_H .$$

It was shown in Ref. 3 that the energy $-(\gamma_m^2/2)\hbar\omega_H$—is essentially the energy of the bound states of an electron in the field of an individual, shallow attracting center in the state with projection of the orbital moment m in the direction of **H**. Thus, the temperature and field dependences give direct information on the energy spectrum of the impurity. Equation (3) has a simple physical meaning: under the integral sign we have the product of the densities of the initial and final states of an electron ϵ^{-1} and the coefficients of transmission of an electron through a shallow well $D_m = (1 + \Delta m/\epsilon)^{-1}$ in the states m and $m+1$, which are interchanged by the electric field in the linear approximation. At low energies the electron wave function is small in the region $\sim \hbar p^{-1} \gg a$ and the conductivity vanishes.

We should mention one more effect which is observed in semiconductors with ellipsoidal $m^* = \{m_\parallel, m_\perp\ m_\perp\}$, isoenergetic surfaces such as Ge and Si, according to Eq. (2). If the direction of H changes relative to the crystallographic axes, the parameter $m^* U a^2/\hbar^2$, which characterizes the depth of the well, will change from $m_\parallel U a^2/\hbar^2$ to $m_\perp U a^2/\hbar^2$ as a result of changing the direction of **H** by $\pi/2$. In Ge the parameter of the anisotropy of masses $m_\parallel/m_\perp \approx 20$. Zero energy appear levels and the reflection coefficient oscillates as a result of changing the depth of the well, which leads to oscillations of σ.

I am very grateful to Yu. A. Gurvich and M. A. Kozhushner for discussions of this work.

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