The structure of the vacuum in gauge theories and cosmology

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Arguments are presented showing that the ground state in quantum chromodynamics, rather than being a θ vacuum, is characterized by a density matrix, which implies averaging over all permissible values of θ. The result is that strong *CP* violation is automatically missing in the theory.

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In recent years, it has been determined that the vacuum in quantum chromodynamics and in other gauge theories has an unexpectedly rich structure.[1] In particular, it was found that "vacuums," which are related to a theory with the same Lagrangian, may differ from each other by a certain parameter θ that characterizes them (see below), and which theoretically may have arbitrarily high values. On the other hand, according to Peccei and Quinn,[2] the parameter θ, which exceeds in the modulus $\sim 10^{-8}$, would lead to unacceptably strong violation of the *CP* invariance in most of the currently used theories of weak, strong, and electromagnetic interactions. Generally, there are a number of models which do not have a strong *CP* violation at arbitrary values of θ (these are primarily models with axioms[3]). At present, however, all of these models seem to be either insufficiently simple and natural to be realistic or clearly contradict the experimental data.

In this paper we would like to present certain arguments showing that if the parameter θ can be nonvanishing, then the ground state in quantum chromodynamics is actually not a θ vacuum but is characterized by a density matrix, which implies averaging over all possible θ. As a result of this averaging, all the effects of strong CP violation in the gauge theories vanish.

First, we call to mind, in keeping with Refs. 1 and 4, certain facts concerning the structure of the vacuum in gauge theories, using as an example the SU(2) symmetric theory of a massless Yang-Mills field A_μ^α with the Lagrangian

$$L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} .$$

Here $\alpha = 1, 2, 3$ is the isotopic index

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc} A^b_\mu A^c_\nu .$$

We shall work in the gauge $\frac{1}{2} A^a_0 \tau^a \equiv A_0 = 0$, in which there is still freedom for time-independent gauge transformations

$$\frac{1}{2} A^a_i \tau^a \equiv A_i \to U^{-1}(\lambda)\, A_i\, U(\lambda) \,, \tag{1}$$

where

$$U(\lambda) = e^{-i Q(\lambda)} = e^{-i \int E^a_i(\mathbf{x})\,(D_i \lambda(\mathbf{x}))^a d^3x} . \tag{2}$$

Here $E^a_i = \partial_0 A^a_i$ is the color electric field,

$$(D_i \lambda)^a = \partial_i \lambda^a + \epsilon^{abc} A^b_i \lambda^c .$$

The vacuum state, which we denote by $|\theta\rangle$, must satisfy Gauss's theorem $(D_i E_i)^a |\theta\rangle = 0$; taking this theorem into account, it follows from Eq. (2) that

$$e^{-i Q(\lambda)} |\theta\rangle = e^{-i \int E^a_i \lambda^a ds_i} |\theta\rangle \,, \tag{3}$$

where the integral in Eq. (3) is taken over the surface at infinity, which encompasses all the space. If the $\lambda^\alpha(\mathbf{x})$ function does not decrease at infinity, then the vacuum state undergoes a transformation under gauge transformation (3). Specifically, as a result of a "single-instanton" transformation $\lambda^a_1(x) = 2\pi \mathbf{x}^a / \sqrt{(\mathbf{x}^2 + \rho^2)}$, the vacuum vector can rotate by a certain angle—θ [1,4]:

$$e^{-i \int E^a_i \lambda^a_1 ds_i} |\theta\rangle = e^{-i\theta} |\theta\rangle . \tag{4}$$

According to Refs. 1 and 2, the parameter θ is responsible for strong violation of CP invariance in the theory.

It can be seen from Eq. (4) that the possible difference of θ from zero is purely a boundary effect.[1)] Thus, the question of the boundary conditions in the real universe arises.

The answer to this question is rather unexpected: in principle, we cannot have any information on the boundary conditions in the universe, which account for the surface integral in Eq. (4). The reason for this is that we can observe only a limited part of the universe inside a sphere of radius $R = ct \sim 10^{28}$ cm, where t is the existence time of the universe and R is the so-called horizon radius. In this case the "ignorance principle," which was first introduced by Hawkings in connection with the theory of evaporation of black holes, comes into force[7]: to allow for the existence of the horizon, after the calculations, an averaging with a unit weight should be performed over all possible states of the system beyond the horizon. This means that in the presence of a horizon the ground state is no longer a pure (vacuum) state but is characterized by a certain density matrix, which, in particular, leads to the existence of blackbody radiation from the surface of the horizon (Hawkings effect[7]).

In our case the averaging with a unit weight over all possible states of the system beyond the horizon also indicates averaging of the scattering amplitudes over all the boundary conditions, i.e., over all θ after performing calculations in different θ vacuums. Thus, it seems that the averaging should be performed with the weight $f(\theta)$, which is independent of the sign of θ.

This conclusion, which, it seems to us, is of interest in itself, is directly related to strong CP violation. In fact, the strong CP violation is attributed to the fact that the following CP noninvariant term appears in the effective action after taking into account the nontrivial structure of the vacuum

$$-\frac{\theta}{32\pi^2} \int F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}\, d^4x,$$

where $\tilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F^a_{\alpha\beta}$. After averaging over θ the generating functional of the Green's functions in the θ vacuum $Z_\theta(J)$[1,2] is replaced by

$$Z(J) = \int_{-\infty}^{\infty} Z_\theta(J) f(\theta)\, d\theta .$$

If no additional CP- violating terms $\sim \int F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} d^4x$ except the $(\theta/32\pi^2)$ $\int F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} d^4x$ term is added by "hand" to the action, then the CP transformation in the theory is equivalent to the substitution $\theta \to -\theta$ relative to which the quantity $Z(J)$ [in contrast to $Z_\theta(J)$] is invariant.

Moreover, in a separate publication we shall present arguments showing that if $\theta = 0$ is not a uniquely possible parameter θ, then the averaging over all θ must be performed with a unit weight:

$$Z(J) = \int_{-\infty}^{\infty} Z_\theta(J)\, d\theta .$$

In this case, after integration over θ $\delta(\int F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} d^4x)$, appears as a factor under the integral sign in the functional integral for $Z(J)$, which leads to the disappearance of all the dangerous terms $\sim \int F^a_{\mu\nu} \tilde{F}^a_{\mu\nu} d^4x$, thereby ensuring automatic absence of strong *CP* violation in the gauge theories.

A more detailed discussion of the problems touched upon above will be included in a separate publication. In conclusion, the author would like to take the opportunity to thank B. L. Voronov, R. É. Kallosh, D. A. Kirzhnits, R. Kryuzer, A. M. Polyakov, I. V. Tyutin, V. Ya. Faĭnberg, V. P. Frolov, A. S. Shvartz, and M. A. Shifman for frequent discussion of the problems connected with the structure of vacuum in the gauge theories and cosmology.

[1)]The influence of boundary effects on violation of *CP* invariance in quantum chromodynamics in the confinement phase, in which the range of forces does not exceed 10^{-13} cm, is not completely clear yet.[5] It could be assumed that generally the results of Ref. 2 are only a consequence of the approximation adapted for studying the phase of the free quarks with long-range forces between them, rather than for studying the confinement phase (see, however, Ref. 6).

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