Theory of anomalous Hall effect of spin glass

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Theory of anomalous Hall effect (AHE) of spin glass is formulated in the molecular field approximation. The temperature dependence of the AHE constant is obtained with allowance for the Kondo effect. The AHE constant is shown to be independent of impurity concentration c if $c \ll 1$.

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Among the kinetic effects the anomalous Hall effect (AHE) is the most sensitive to magnetic phase transitions.[1] It should be expected, therefore, that by studying the temperature and concentration dependence of the AHE in spin glass can help to explain the magnetic state of the latter. In this paper we report the results of calculations of the AHE in spin glass in the molecular field (MF) approximation.

We express the Hamiltonian of the problem in the following form:

$$\hat{H} = \sum_k \epsilon_k a_k^+ a_k - \sum_n g\mu_B H_n (S_n)z_n - \sum_n g\mu_B H^z S_n^z + H_{int}, \tag{1}$$

$$\hat{H}_{int} = \sum_n (V_n)_{kk'} a_k^+ a_{k'} - \sum_n J(\vec{\sigma} \mathbf{S_n}) + \sum_n \lambda a^2 [\mathbf{k}\,\mathbf{k}']^z S_n^z . \tag{2}$$

The first three terms in Eq. (1) describe the energy of the conduction electrons, the interaction between the magnetic impurities in the MF approximation, and the interaction between the impurities and the external magnetic field, respectively. H_{int} represents the sum of the Coulomb, exchange, and spin-orbit coupling (SOC) of the conduction electrons with randomly distributed magnetic impurities (a is the lattice parameter and λ is the SOC constant). The SOC produces asymmetry in the conduction-electron scattering, which causes the the AHE. The external magnetic field H^z is directed along the O_z axis of the laboratory coordinate system, whereas the local field H_n acting on the nth impurity is directed along the O_{z_n} axis of the local coordinate system determined by the spin direction of the nth impurity at $T = 0$. The random distribution of the local O_{z_n} axes determines the vanishing of the spin-glass magnetization.

The AHE constant R_a is determined by the relation:

$$R_a = \frac{\sigma_{xy}^{(1)}}{4\pi \sum_n \langle S_n^z \rangle [\sigma_{xx}^2 + \sigma_{yx}^2]} \approx \frac{\sigma_{yx}^{(1)}}{4\pi \sum_n \langle S_n^z \rangle \sigma_{xx}^2}, \tag{3}$$

where $\sigma_{xy}^{(1)}$ is SOI linear anti-symmetrical part of the electrical conductivity tensor. The calculation of R_a reduces to the following. Using the kinetic Boltzmann equation, we determines the expression relating σ_{xx} and σ_{yx} to the corresponding probabilities

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$w^{(0)}$ and $w^{(1)}$. Assuming that we are dealing with the Born scattering ($J/E_F \ll 1$, $V/E_F \ll 1$) and the probabilities of scattering by different impurities are additive ($w = \Sigma_n w_n$), we calculate $w_n^{(0)}$ and $w_n^{(1)}$ in the local coordinate system associated with the nth impurity. In our calculations we used a diagram technique similar to that described in Ref. 2. Moreover, the calculation of $w_n^{(0)}$ was carried out to the third order, and $w_n^{(1)}$, the fourth order with respect to the scattering potential. In the latter orders we retained only the λJ^3-type terms that determine the Kondo effect. The thermal averaging of spin correlations in the expressions for the scattering probabilities and the averaging with respect the local fields H_n were carried out in the local coordinate system. For simplicity, we assumed that the local fields are distributed uniformly in the interval $[-\Delta,\Delta]$; thus, the average value of the arbitrary vector function $\mathbf{f}(g\mu_B H_n)$ is

$$< \mathbf{f}(g\mu_B H_n) > = \frac{g\mu_B}{2\Delta}\int_{-\Delta}^{\Delta} \mathbf{f}(g\mu_B H_n)\,dH_n\,, \tag{4}$$

$$\Delta = g\mu_B \sqrt{q} H_0\,.$$

Here q is the temperature-dependent order parameter of the spin glass[3] and H_0 is directly related to the freezing point of the spin glass T_f by the relation $g\mu_B H_0 = m\kappa T_f$ (κ is the Boltzmann constant), where m is a numerical coefficient that depends on the impurity spin. For $S = 1/2$ $m = 2\sqrt{3}$. Although all the calculations were performed for arbitrary values of S and H^z, below we give results only for $S = 1/2$ and $g\mu_B H^z \ll \kappa T_k$ (T_k is the Kondo temperature).

The final expression for R_a has the following form:

$$R_a = \lambda A[V^2 + J^2 B_1(T, S) + J^3 B_2(T, S)], \tag{5}$$

where the constant A is independent of the impurity concentration c and of the temperature. For spin $S = 1/2$ at $T > T_f$ and $T \lesssim T_f$ the coefficient B_1 is independent of temperature and is equal to $\frac{1}{4}$, and at $T_k \ll T \ll T_f$ $B_1 = (\frac{1}{4})\,[1 - (\kappa T/E_F)e^{-\Delta/\kappa T}]$. At low temperatures the exponential temperature dependence is associated with a gap in the excitation spectrum, which occurs in the MF approximation, and can vary if we take into the account the possibility that spin waves exist in the spin glass. We should emphasize that the use of the perturbation theory in the calculation of $w^{(0)}$ and $w^{(1)}$ imposed constraints on the use of the obtained results by the condition $T \gg T_K$.

We shall now examine the last term in Eq. (5). At $\Delta/\kappa T < 2$ it has the following form:

$$B_2\left(T, S = \frac{1}{2}\right) = \frac{3n}{E_F} \ln \frac{\kappa T}{2E_F}\,, \tag{6}$$

where n is the concentration of the conduction electrons, and at $\Delta/\kappa T > 10$

$$B_2\left(T, S = \frac{1}{2}\right) = \frac{3n}{E_F} \ln \frac{\Delta}{2E_F}\,. \tag{7}$$

In the limit $\Delta \to 0$, i.e., for the conventional Kondo systems, $B_1(T,S) = \text{const}$, and $B_2(T,S)$, like in the spin glass, contains a logarithmic term.

As seen in Eq. (5), R_a is independent of the impurity concentration at $c \ll 1$. This is attributable to the fact that $\sigma_{xy}^{(1)}/\sigma_{xx}^{(2)} \sim w^{(1)} \sim c$, but the magnetization is proportional to c [see Eq. (3)]. Since the Hall emf is

$$E_H = R_0 H^z (1 + 4\pi\chi) + R_a 4\pi\chi H^z,$$

where χ is the susceptibility, its anomalous part depends linearly on c. The maximum $E_H(T)$ at $T = T_f$, which was observed experimentally in Ref. 4, is related to the maximum $\chi(T)$. To determine experimentally the temperature and concentration dependences of the AHE constant and to compare the proposed theory with the experiment, it is necessary, in addition to measuring the emf in weak magnetic fields and separating the anomalous contribution from the normal contribution to the emf to determine the susceptibility for the same samples under the same conditions, which heretofore has not been done. Such investigations help to refine the nature of spin glass and the mechanisms responsible for the AHE.

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