Relaxation of longitudinal magnetization in the ^3He A-phase and superfluid spin flux

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Superfluid spin flux in the A phase of ^{3}He is possible only if the energy that holds the spin vector **d** in the perpendicular plane to the magnetic field exceeds the dipole-dipole energy. The effect of superfluid spin flux on the relaxation rate of the longitudinal magnetization under different conditions at the boundary of the investigated ^{3}He is disscussed.

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A number of papers has been devoted to experimental and theoretical investigation of the relaxation process of the longitudinal magnetization in superfluid ^{3}He. The relaxation occurs much more rapidly than in the normal phase, and frequently has an unusual nonlinear character. Corruccini and Osherov[1] observed for the first time a linear decrease in the magnetization with time. They attributed this behavior to the formation of large, critical, superfluid spin flux from the volume in which the magnetization relaxation was studied. This relaxation mechanism was studied theoretically by Vuorio.[2] The present communication is devoted to a further analysis of this mechanism. First we shall show that the superfluid spin transport can occur only in magnetic fields exceeding a certain characteristic value determined by the equality of the dipole-dipole energy and the interaction energy with the magnetic field.

We shall examine the relaxation of the spin magnetization along the constant magnetic field **H** (z axis) in the A phase. If the spin vector **d** does not leave the xy plane, then the state of the system is defined by a pair of canonically conjugate variables[3]: the spin density S_z along the z axis and the rotation angle ϕ of the vector **d** in the xy plane relative to the orbital vector **l**, which is assumed to be fixed and lying in the xy plane. The Hamiltonian equations have the form

$$\frac{\partial S_z}{\partial t} = -\operatorname{div} \mathbf{j} + \frac{1}{2} G_D \sin 2\phi , \tag{1}$$

$$\frac{\partial \phi}{\partial t} = - \frac{\gamma^2 S_z}{\chi} , \tag{2}$$

where S_z is the deviation of the spin density from the equilibrium value $\chi H/\gamma$, γ is the gyromagnetic ratio, and χ is the susceptibility. The expression for the spin current

$$\mathbf{j} = A \nabla \phi - D \nabla S_z \tag{3}$$

includes a dissipative spin diffusion flux $D\nabla S_z$, in addition to the superfluid flux (spin "superflux") $A\nabla\phi$, which is determined by the rigidity of the order parameter A.

If the **d** vector lies in the plane, the spin superflux is dissipation-free, which is attributed to the stability (metastability) of the helicoidal texture with nonvanishing $\langle \nabla\phi \rangle$, i.e., spatial rotation of **d** in the xy plane. At small average gradients

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$\langle\nabla\phi\rangle \ll L_D^{-1}$ ($L_D = \sqrt{A/G_D}$ is the size of the domain wall), the texture is a periodic structure of **d** solitons,[4] which are 180-degree domain walls between the domains with oppositely directed **d** vectors. However, the texture with nonvanishing $\langle\nabla\phi\rangle$ is not topologically stable (Volovik and Mineev[5]), if **d** can leave the plane. On the other hand, if the xy plane is a "light plane," so that the emergence of the **d** vector from it is accompanied by an increase in the volume energy, then the destruction of the domain walls (they must be destroyed in pairs) occurs by overcoming a certain energy activation barrier, which is determined by the energy of the "vortex" line along which the **d** vector leaves the xy plane, and established perpendicular orientation to it; as a result of rotating around it, the angle ϕ increases by 2π. Such an activation barrier was calculated in Ref. 6 for an analogous problem of stability of the helicoidal structure in a light-plane antiferromagnet. In the ^{3}He A phase the **d** vector is oriented by the magnetic field in the xy "light plane" (the interaction energy is $\sim\Delta\chi H^2$, where $\Delta\chi$ is the difference of the susceptibilities in the transverse direction to **d** and along it), and the dipole-dipole energy $\sim G_D$ plays the same role as the anisotropy energy in the light plane of the antiferromagnet, which destroys the degeneracy of the angle ϕ. Using the results of Ref. 6, we may immediately obtain an expression for the activation barrier (the vortex line is assumed to be a half-circle lying on the boundary of the system):

$$E_b = \begin{cases} \dfrac{\pi^3}{8\sqrt{2}} A L_D \left(\ln \dfrac{L_D}{r_C}\right)^2 & \langle\nabla\phi\rangle \ll \dfrac{1}{L_D} \\[2ex] \dfrac{\pi^2}{4} \dfrac{A}{\langle\nabla\phi\rangle}\left(\ln \dfrac{1}{r_C\langle\nabla\phi\rangle}\right)^2 & \langle\nabla\phi\rangle \gg \dfrac{1}{L_D} \end{cases} \tag{4}$$

Here r_C is the size of the core of the "vortex," i.e., the distance from the vortex line on which the "rigidity" energy $A(\nabla\phi)^2 \sim A/r_C^2$ is of the same order as the interaction energy with the magnetic field $\Delta\chi H^2$.

The activation barrier can reach large values if $L_D \gg r_C$, i.e., if the field H exceeds a certain value H_c determined by the condition $\Delta\chi H_c^2 \sim G_D$, and is equal to 30 G, according to Ref. 7. This conclusion is independent of the texture of the vector **l** and agrees with the results of experiments,[8] in which for fields of 30–85 G a transition was observed from the Leggett-Takagi[9,10] volume mechanism for relaxation to a linear relaxation law associated, according to Corruccini and Osherov,[11] with the superfluid spin transport.[1)]

We shall now examine how superfluid spin transport at $H > H_c$ affects the process of ^{3}He superfluid relaxation in a layer $-d < x < d$. The medium with a purely diffusive spin propagation occupies the space $|x| > d$, but with a rather strong spin relaxation source (Corruccini and Osherov[11] assumed that the paramagnetic impurities on the walls are such a medium). Thus, Eqs. (1) and (2) (we shall disregard the dipole-dipole interaction and assume that the magnetization and spin flux are sufficiently large) should be solved with a boundary condition for the spin flux at $x = \pm d$:

$$j = \pm v_0 S_z \qquad v_0 = \sqrt{D'/T_1}\ , \tag{5}$$

where D' is the diffusion coefficient and T_r is the Block relaxation time in the region $|x| > d$. Solving this boundary problem in the limit $v_0 \to 0$, we can see that the magnetization dies away exponentially with relaxation time $T_r = d/v_0$, irrespective of whether the spin transport in the region $|x| < d$ is attributable to the diffusion or to the superflux. However, the limits of applicability of this relaxation law are basically different for the two cases. If the spin is transported by a superflux, then $T_r = d/v_0$ up to the values of v_0 of the order of the spin waves $\gamma\sqrt{A/\chi}$. For a pure diffusion [$A = 0$ in Eqs. (1)–(3)] this is valid only to $v_0 \ll D/d$, but at $v_0 \gg D/d$ the solution of the problem gives the relaxation time $T_r = 4d^2/\pi^2 D$, which can greatly exceed the time d/v_0.

The so-called open geometry is often studied in experiments, in which the investigated (volume) inside the current coil) is not separated from the remaining volume of the superfluid ^{3}He, and a removal of the spin to a considerable distance without absorption is assumed to be the relaxation mechanism. In this case, since the magnetic field outside the coil decreases and, according to what was said above, the dissipation-free transport of the spin by the superflux is impossible, the spin transport just outside the range of the field of the coil becomes the "narrow place" of the relaxation process; however, the superfluid transport of the spin inside the volume has little effect on the relaxation rate. To illustrate this, we shall modify somewhat the problem of relaxation discussed above. Let us assume that in the region $|x| > d$ there is a pure diffusion without absorption ($T_1 \to \infty$), and the region $|x| < d$ has an initial nonequilibrium spin density S_0, and the spin transport to the boundary via the superflux is rapid, so that $S_z = \text{const}$ for $|x| < d$ and for $x = \pm d$ there is a boundary condition $d\dfrac{\partial S_z}{\partial t} = \mp j$. Solving the diffusion problem in the region $|x| > d$ with these boundary and initial conditions, we obtain

$$S_z = S_0 \exp\left(\frac{D't}{d^2} + \frac{|x| - d}{d}\right) erfc\left(\frac{|x| - d}{2\sqrt{D't}} + \frac{\sqrt{D't}}{d}\right). \tag{6}$$

If, however, the spin is transported by diffusion in the region $|x| < d$, then:

$$S_z = S_0 \frac{1}{2}\left[erfc\left(\frac{|x| - d}{2\sqrt{D't}}\right) - erfc\left(\frac{d + |x|}{2\sqrt{D't}}\right)\right]. \tag{7}$$

Both expressions give $S_z \approx S_0\, d/\sqrt{\pi D't}$ for large times $t \gg d^2/D'$.

To take into account the dissipation due to the vortices, a "frictional force" must be introduced into Eq. (2). If we assume that this force is missing below a certain critical value $\nabla\phi$ and then begins to increase rapidly, we immediately obtain the observed linear spin relaxation law. A direct observation of the onset of "spin turbulence," i.e., a large number of vortices, in the neighborhood of which the vector **d** leaves the "light plane" would confirm the existence of a large spin superflux during its relaxation. The observed nonmonotonic spin relaxation[8,11] is sometimes considered as such confirmation. However, other explanations of this effect have also been suggested.[10]

[1)]A strong dependence of the relaxation on H and the Liggett-Takagi relaxation at weak H were not observed; however, in Ref. 8 in the so-called "horizontal geometry," where the ^{3}He boundary layer was oriented normal to the field **H** and oriented the vector **1** along **H**, eliminating thereby the violation of the symmetry in relation to the rotation around **H** without which the Leggett-Takagi mechanism is suppressed, was observed.

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