Relativistic effects and parity violation in the radiative capture of a neutron by a proton

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It is shown that the isovector contributions to the circular polarization of photons in the $n\,p \to d\gamma$ capture, which are attributed to the relativistic effects (primarily the off-mass effects, and retardation and mobility of the nucleon magnetic moments) are large in the Weinberg-Salam model of electroweak interaction.

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An interest in the search for relativistic effects (RE) in the interaction of particles with nuclei has increased in recent years (see review article[1]). Until recently, RE occurring in the process with parity violation have been studied very little.[1)] It was shown[4] that the relativistic contributions to the circular polarization of photons in the P_γ $n\,p \to d\gamma$ capture of slow neutrons, which depend on the off-mass effects and change the nonrelativistic selection rules with respect to isospin for the weak interaction, can be important in the models with neutral currents, especially in the Weinberg-Salam model of electroweak interaction for which the weak π-meson-nucleon coupling constant is large ($g^W_{\pi NN} = (0.3\text{–}0.8)\times 10^{-6}$).[5–7]

The variation of nonrelativistic isospin selection rules in the case of circular photon polarization in the reaction $n\,p \to d\gamma$, allowing for the off-mass effects, can be explained as follows. As is well known, the amplitude of the exchange by a single (or several) π meson with parity violation is an isovector, and it does not change the total spin for the nucleons on the mass surface. For nucleons outside the mass surface, an exchange by an arbitrary number of π mesons produces an amplitude which is also an isovector but which changes the spin of the np system, for example,

$$M^{(m\pi)}_{np} = C^{(m\pi)} f_{m\pi}(\Delta)(p_1^2 - p_2^2)\,[(\vec{\sigma}_1 - \vec{\sigma}_2)\ (\mathbf{p}_1 - \mathbf{p}_2)\,]\,T^{-}. \qquad (1)$$

Here $T^{-} = [\vec{\tau}_1\vec{\tau}_2]_3$, $\vec{\sigma}_{1,2}$, $\tau_{1,2}$ are the Pauli matrices of the nucleon spin and isospin, $p_{1,2}$ are the momenta of both nucleons in the initial (or final) state, Δ is the transferred

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momentum, and $C_{np}^{(m\pi)}$ is a certain constant that includes the π meson-nucleon coupling constants. The amplitude (1) is proportional to the small multiplier $p_1^2 - p_2^2$—difference of the squares of the nucleon 4-momenta, and vanishes for the nucleons on the mass surface. Otherwise, the Pauli principle would be violated for nucleons in the initial or final states. The calculation of this effect is cumbersome in the nonrelativistic theory and is most suitable in the framework of the field theory approach,[4] in which the deuteron and the scattering states are described by the Bethe-Salpeter vertices which, by virtue of the generalized Pauli principle, satisfy certain symmetry relations. The amplitude of the process $np \to d\gamma$ with parity violation has the following (explicitly gauge-invariant) form:

$$M^-_{a\nu} = e\int\left\{\frac{(d+d')_a}{2dq}\left[B_\nu(d', P, Q\ldots) - q_\nu\frac{B_\mu(d', P, Q\ldots)d'_\mu}{d^2}\right]\right.$$
$$+\frac{1}{q_a}[B_\nu(d, P, Q\ldots) - B_\nu(d', P, Q\ldots)] + \delta_{a\nu}\frac{B_\mu(d', P, Q\ldots)d'_\mu}{d^2}$$
$$\left.-\frac{(d+d')_a}{2d'q}B_\nu(d, P, Q\ldots)\right\}\frac{d^4P\,d^4Q\ldots}{(2\pi)^8(P^2-M^2+i\delta)(Q^2-M^2+i\delta)} \qquad (2)$$

where M is the nucleon mass, a and ν are the photon and deuteron indices, respectively, P, Q, etc. are the 4-momenta of the integration in the intermediate states, and d', d, $q = d' - d$ are the momenta of the initial state of the deuteron, and of the photon. In this expression the contact terms were reconstructed from the condition of gauge invariance. "Division" by q_a denotes differentiation, if B_ν does not contain a pole. Otherwise,

$$\frac{1}{q_a}\left[\frac{1}{(d-P)^2-M^2} - \frac{1}{(d'-P)^2-M^2}\right] = \frac{(d'+d-2P)_a}{[(d'-P)^2-M^2][(d-P)^2-M^2]}.$$

B_ν may correspond to the exchange by an arbitrary number of the vector and pseudoscalar mesons (Fig. 1). In the case of the exchange by ρ and ω mesons ($\Delta T = 0$), the result of nonrelativistic calculation with the electric dipole transition operator $\omega\mathbf{r}$ is

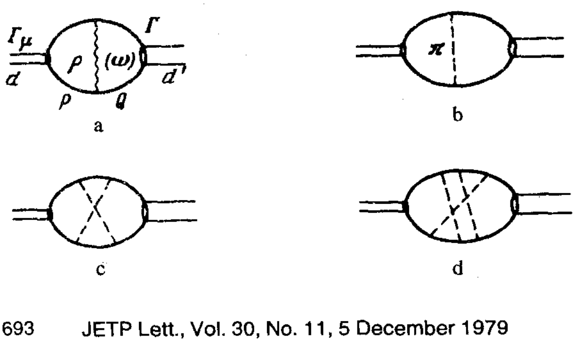

FIG. 1.

reproduced from Eq. (2).[8] In the case of the exchange by π mesons, B_ν contains a small factor $(d-P)^2-P^2$ or $(d-Q)^2-Q^2$, and this reflects the nonrelativistic exclusion of the isovector contributions to P_γ. For the pseudo-scalar (strong) π meson-nucleon coupling, B depends on the so-called relativistic components of the initial or final states.[4,9] In the case of pseudo-vector πNN coupling,

$$B_\nu = (8M/\mu_\pi) f g^W_{\pi NN} \{ AF\,2Md(P-Q)(P+Q)_\nu + AGP_\nu[d(P-Q)(dP+dQ - P^2 - M^2 - 2PQ) + Q(P-Q)(2Pd-d^2)]\}\{[(d-P)^2-M^2][(d-Q)^2-M^2] \times [(P-Q)^2-\mu_\pi^2]\}^{-1}, \qquad f^2/4\pi = 0.16. \tag{3}$$

Here, the vertex functions of the deuteron and the scattering states are used in the form $\Gamma_\mu = F\gamma_\mu + GP_\mu$, $\Gamma = A\gamma_5$; moreover, $F(P,d-P) = F(d-P,P)$ and, analogously, for A and G due to the aforementioned symmetry properties. F and A describe the S waves and G describes the D wave in the deuteron.

The contribution to $M_{a\nu}$ from a part of the $B_\nu \sim (P+Q)_\nu\ (2dQ-d^2)$ can be converted to the following form (in the final answer, we can set $d'=d$):

$$M_{a\nu} = \frac{8M^2}{\mu_\pi} e f g^W_{\pi NN} \int \frac{AF d^4P d^4Q/(2\pi)^8}{(P^2-M^2)[(d-P)^2-M^2][(P-Q)^2-\mu_\pi^2]}$$
$$\times\left\{\left[\frac{1}{Q^2-M^2} - \frac{1}{(d-Q)^2-M^2}\right]\left[\delta_{a\nu}\left(\frac{dP+dQ}{d^2}-1\right) + (P+Q)_\nu P_\alpha \right.\right.$$
$$\left.\left.\times\left(\frac{1}{P^2-M^2} - \frac{1}{(d-P)^2-M^2}\right)\right] + (P+Q)_\nu Q_\alpha\left[\frac{1}{(Q^2-M^2)^2} + \frac{1}{((d-Q)^2-M^2)^2}\right]\right\}. \tag{4}$$

This expression does not vanish as a result of integration with respect to d^4Pd^4Q, since the function under the integral sign is symmetrical with respect to the substitution $P \to d-P$, $Q \to d-Q$. For simplicity, we assumed that $A = A(Q-d/2)$, and F, $G = F, G(P-d/2)$. It should be noted that in calculating Eq. (4) we should not limit ourselves to the contribution from only the nearest singularities in P_0 and Q_0, in contrast to the nonrelativistic case (vector meson exchange). The singularities of the π-meson propagator and of the vertex functions are equally important in Eq. (4). Therefore, to calculate Eq. (4), we must know the structure of the singularities A, F, etc. in the complex energy plane. If, for example,

$$A(F,G) \sim \frac{a^2}{\left(Q-\frac{d}{2}\right)^2 - a^2 + i\delta},$$

then the standard calculation of the Feynman integrals is possible by using the Wyckoff rotation.

The order of magnitude of the effect for a single-pion exchange with the pseudovector coupling is

$$|P_\gamma| \sim \frac{2M}{\mu_\pi} f g^W_{\pi NN} \frac{M a_s < P^4/M^4 >}{4\pi(\kappa a_s - 1)(\mu_p - \mu_n)} \approx 0.1\, g^W_{\pi NN}, \quad \kappa^2 = M\omega,$$

$$\mu_p - \mu_n = 4.7 .$$

The 2π-exchange with the pseudo-scalar coupling leads to a noticeably larger result. An unambiguous answer is difficult to obtain, since the contributions from the exchanges by a different number of π mesons must be summed, the form factors at the πNN vertices must be taken into account, and the relation between the pseudo-vector and pseudo-scalar constants of the πNN coupling must be known. The experimental value of P_γ[10] must also be verified. It is clear, nonetheless, that the isovector contributions to P_γ can be dominant, if there is no reason to compensate for the integrals of the form (4).

The effect under consideration is associated with the violation of the Siegert theorem,[11] since it cannot be calculated in terms of the standard approach with the operator $\omega\mathbf{r}$. The question concerning the violation of the Siegert theorem while conserving the parity has been widely discussed in recent years.[12-14] The case of spatial parity violation is particularly glaring: violation of the Siegert theorem is large here due to the relativistic effects. This can be explained as follows. If we examine Eq. (1) in terms of an addition to the effective interaction Hamiltonian and perform the standard substitution: $p_i \to p_i - eA(r_i)(1+\tau_{3i})/2$, then we obtain an addition to the electromagnetic current operator $J_\nu \sim eT^-\ (p_1 - p_2)_\nu\ (\vec{\sigma}_1 - \vec{\sigma}_2)\ (\vec{\mathbf{p}}_1 - \vec{\mathbf{p}}_2)$, which is an isovector, and is not reducible to the Hamilton commutator $[H, (\tau_{13} - \tau_{23})\mathbf{r}_{12}]$, $\mathbf{r}_1 - \mathbf{r}_2$ and is quadratic with respect to the relative momentum of the nucleons. This argument is only illustrative, and the calculations should be performed in accordance with Eq. (2), etc.

In addition to the off-mass effects and isovector contribution to P_γ, the motion of the nucleon magnetic moments and retardation (Fig. 2) should also be taken into account (this leads to appearance of the spin-dependent terms in the electric-dipole-transition operator). These effects, however, are less important: the contribution due to retardation contains the factor ω/μ_π—the ratio of the retardation time $1/\mu_\pi$ to the characteristic time of the process $1/\omega$.

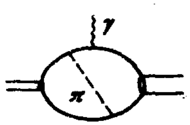

FIG. 2.

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