The smallness of cross sections for scattering of resonances and particle beams by nucleons

V. S. Verebryusov and L. A. Ponomarev
Institute of Theoretical and Experimental Physics

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The contribution to the resonance vertex function of the real states of decaying particles was determined. It is shown that allowance for this contribution can be used to explain the basic characteristics of the cross sections for scattering of resonances and particle beams by nucleons.

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1. We shall examine the general properties of the vertex functions (VF) of resonances and use the obtained results in the analysis of experimental data for the integral cross sections of scattering of resonances and particle beams by nucleons, which were obtained as a result of analyzing the experiments on the production of these systems by nuclei.[1)]

For simplicity, we examine a scalar VF of the S-wave resonance that decomposes into two scalar particles a_1 and a_2 with masses μ_1 and μ_2. We denote the mass and width of the resonance by M and Γ ($M>\mu_1+\mu_2$). We denote by p_1 and p_2 the 4-momenta of the resonance before and after the interaction, $k=p_1-p_2$, $s_i=p_i^2$; we represent the resonance VF as $G(s_2,s_1,k^2)$. The contribution of the latter to the single-scattering amplitude can be expressed as follows (Fig. 1a):

$$\Pi(s_2)G(s_2,s_1,k^2)\Pi(s_1), \tag{1}$$

where $\Pi(s)=-\left[s-M^2+i\sqrt{s}\,\Gamma(s)\right]^{-1}$ is the resonance Green function, $\Gamma(s)=\lambda^2Q/8\pi s$, $Q=\sqrt{[s-(\mu_1+\mu_2)^2][s-(\mu_1-\mu_2)^2]}\,/2\sqrt{s}$, and λ is the VF of the decay interaction.

We use $G_0(k^2)$ to represent the resonance VF in the region of nondecaying masses $[s_1,s_2<(\mu_1+\mu_2)^2]$. In the numerical calculations, we assume that this value is predicted by the quark model.

Assuming that G_0 is weakly dependent on the resonance mass $[G_0=G_0(k^2)]$, we

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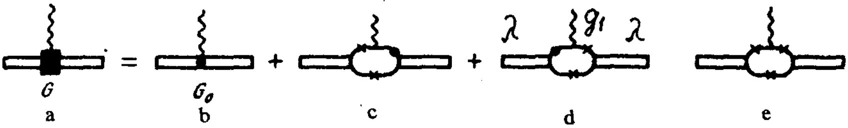

FIG. 1.

calculate the variation of VF on going to the decaying masses of the resonance ($s_1,s_2 \approx M^2$) by using the graphic equation in Fig. 1, which includes all the diagrams that contain the real states of decaying particles (we assume that only one particle a_1, is involved in the interaction).[2)]

The lines with crosses in Fig. 1 correspond to Green's functions of particles on the mass surface, and those with dots, correspond to the integral Green's function after the subtraction of the mass surface contribution.

The graphic equation in Fig. 1 corresponds to the following value of VF

$$G(s_2, s_1, k^2) = G_0(k^2) + g_1(k^2)\,[\,i\sqrt{s_1}\Gamma_1 I_{1p} + i\sqrt{s_2}\Gamma_2 I_{2p} + i\sqrt{s_1}\Gamma_1 I_{1\delta}\,], \quad (2)$$

where g_1 is the VF of the particle a_1, $\Gamma_i = \Gamma(s_i)$, and I_{ip} and $I_{i\delta}$ are contributions of the off-mass surface and the mass surface in the integrals

$$I_i = I_{ip} + I_{i\delta} = \frac{-1}{4\pi}\int d\Omega_i\,[(r \mp k)^2 - \mu_1^2 + i\epsilon]^{-1},$$

$$I_{1p} = \frac{-1}{4\pi}\int d\Omega_1 P\left[\frac{1}{-2(rk)+k^2}\right] = \frac{-1}{4Q_1|\mathbf{k}|_1}\ln\left|\frac{2Q_1|\mathbf{k}|_1 - 2E_1\omega_1 + k^2}{2Q_1|\mathbf{k}|_1 + 2E_1\omega_1 - k^2}\right|,$$

$$I_{2p} = \frac{-1}{4\pi}\int d\Omega_2 P\left[\frac{1}{2(rk)+k^2}\right] = \frac{-1}{4Q_2|\mathbf{k}|_2}\ln\left|\frac{2Q_2|\mathbf{k}|_2 + 2E_2\omega_2 + k^2}{2Q_2|\mathbf{k}|_2 - 2E_2\omega_2 - k^2}\right|, \quad (3)$$

$$I_{i\delta} = \frac{-1}{4\pi}\int d\Omega_i\,[-i\pi\delta(\mp 2(rk) + k^2)] = \frac{i\pi}{4Q_i|\mathbf{k}|_i}\Theta(s_1 - z_1)\Theta(z_2 - s_1), \quad (4)$$

$$E_1 = \frac{s_1 + \mu_1^2 - \mu_2^2}{2\sqrt{s_1}};\quad E_2 = \frac{s_2 + \mu_2^2 - \mu_1^2}{2\sqrt{s_2}};\quad \omega_1 = \frac{s_1 - s_2 + k^2}{2\sqrt{s_1}};\quad \omega_2 = \frac{s_1 - s_2 - k^2}{2\sqrt{s_2}}, \quad (5)$$

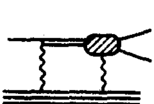

FIG. 2.

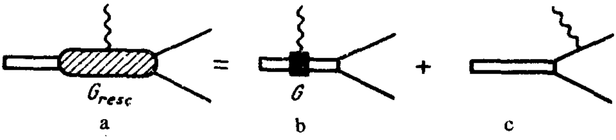

FIG. 3.

where the subscript i corresponds to the cms of the resonance before ($i = 1$) and after ($i = 2$) the interaction, r is the 4-momentum of the particle a, (at the mass surface), ($Q_i\,|\mathbf{k}|_i$), and (E_i, ω_i) are the moduli of the 3-momenta and energy, which correspond to the 4-momenta r and k, and $d\Omega_i$ is the element of the phase exchange. The product of the Θ functions in Eq. (4) limits the region along the s_1 axis, which gives a nonzero contribution of the diagram in Fig. 1e.

$$z_{2,1} = s_2 - k^2\,\frac{s_2 - \mu_1^2 - \mu_2^2}{2\mu_1^2} \pm \sqrt{(-k^2)(4\mu_1^2 - k^2)\Lambda(s_2, \mu_1^2, \mu_2^2)}$$

$$\Lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc. \tag{6}$$

2. In considering the VF corresponding to rescattering of a resonance in any hadron reaction, for example $hA \to RA$ (Fig. 2), it is possible to verify that, within the framework of our approximation taking into account the linear (with respect to the width Γ) terms in the resonance VF, the diagram in Fig. 3c, in addition to those in Fig. 1 must be fully accounted for. We shall define the S-wave state of the final particles in the amplitude corresponding to this diagram

$$T_S = \frac{-\lambda g_1(k^2)}{4\pi} \int d\Omega_2 \frac{1}{(r+k)^2 - \mu_1^2 + i\epsilon} = -\lambda g_1(k^2) I_2 \Pi(s_1). \tag{7}$$

Determining the VF corresponding to this amplitude by the relation $T_s = \lambda \Pi(s_2) G' \Pi(s_1)$ and combining G' with G in (12), we obtain the following expression for an effective VF of the resonance rescattering $G_{\rm resc}$

$$G_{\rm resc} = G_0(k^2) + g_1(k^2)[\,i\sqrt{s_1}\Gamma_1 I_{1p}(s_2, s_1, k^2) - (s_2 - M^2) I_2(s_2, s_1, k^2)]. \tag{8}$$

We shall numerically estimate the Pomeron VF $G_{\rm resc}$ at different k^2, by using for the resonance the ρ-meson parameters $M = 0.76$ GeV, $\Gamma = 0.14$ GeV, and $\mu_1 = \mu_2 = 0.14$ GeV. Since in evaluating the data for production of resonances by

k^2, (GeV/c)²	$-2{,}5\times10^{-3}$	-5×10^{-3}	-10^{-2}	$-0{,}15$
$\overline{F}_R$	$-0{,}53$	0,08	0,45	0,91
$\overline{F}_I$	0,26	0,23	0,34	0,17

nuclei integration is performed with respect to the resonance masses, we shall average G_{resc} in the s_1,s_2 plane at the intersection of the bands $(M-\Gamma/2)^2 < s_1,s_2 < (M+\Gamma/2)^2$. The result of this averaging for the real and imaginary parts G_{resc}, which was divided into the π-meson VF,

$$\bar{F}_R = 2\,\mathrm{Re}\,\overline{G}_{resc}/g_1(k^2), \qquad \bar{F}_I = 2\,\mathrm{Im}\,\overline{G}_{resc}/g_1(k^2) \tag{9}$$

is shown in the Table I. The coefficient "2" in Eq. (9) corresponds to the interaction with the second pion. In constructing Table I, we assumed that the bare value of the Pomeron VF of the ρ meson is equal to the pion VF $G_0(k^2) = g_1(k^2)$, as predicted, for example, by the quark model. Table I demonstrates a strong dependence of the properties of the averaged VF on k^2.

3. We shall use the obtained results in the analysis of the properties of the integral cross sections for rescattering of resonances and particle beams by a nucleon, which were obtained as a result of processing the experimental data for production of these systems by nuclei in the framework of the Glauber approach. For specificity, we examine the ρ meson and the beam (3π).

A. In a series of papers[3-5] it was found that the integral cross section for rescattering of a coherently produced system (3π) is anomalously small. The phase analysis of this system showed that the 1^+ state is characterized by a small rescattering cross section, whereas the 0^- state is characterized by a large one.[6]

$$\text{(a)}\quad \sigma^t_{3\pi,N}(1^+) = 15.8^{+1.5}_{-1.3}\ \text{mb}, \quad \text{(b)}\quad \sigma^t_{3\pi,N}(0^-) = 56^{+14}_{-14}\ \text{mb}$$

$$M_{3\pi} = (1 \div 1.2)\ \text{GeV}, \quad |k^2| < 0.01\ (\text{GeV}/c)^2\,. \tag{10}$$

Interpretation. The (3π) system in the 1^+ state basically consists of a ρ meson and a pion; moreover, the ρ meson and the entire (3π) system are produced in the longitudinally polarized state. It can be shown that the rescattering of a longitudinally polarized ρ meson is close to rescattering of a scalar resonance, and the corresponding VF has the form similar to Eq. (8). It follows from Table I that the real part of the ρ meson VF (which determines the integral cross section) at average transferred momenta characteristic of the coherent peak $[|k^2| \approx (2\text{–}5)\times 10^{-3}\,(\text{GeV}/c)^2]$ is negative or close to zero $\mathrm{Re}\,G_\rho \leqslant 0$. Consequently, the corresponding effective integral cross section of the ρ meson satisfied this condition $\sigma^t_{\rho,N} \leqslant 0$, and the rescattering cross section of the entire system $(\rho\pi)$ satisfied the inequality

$$\sigma^t_{3\pi,N}(1^+) = \sigma^t_{\rho,N} + \sigma^t_{\pi,N} \lesssim \sigma^t_{\pi,N}\,. \tag{11}$$

Assuming that the S-wave state of a pion pair in the region of small masses has basically a nonresonant origin, we can see that the cancellation effect for the cross section $\sigma^t_{3\pi,N}(0^-)$, is missing, so that it should have a "normal" value [Eq. (10b)].

B. A large rescattering cross section $(\sigma^t_{\rho,N} \approx \sigma_{\pi,N})$ was obtained in the experiments in the incoherent production of longitudinally polarized ρ mesons by nuclei.[7]

Interpretation. A characteristic momentum transfer for incoherent production $|k^2|_{char} \approx (0.1\text{–}0.2)(\text{GeV}/c)^2$. As seen in Table I, the contribution for the diagrams 1c,

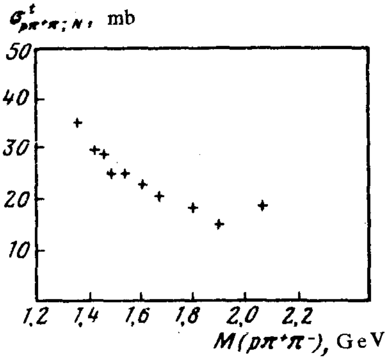

FIG. 4.

1d and 1e is small at this momentum transfer and the VF is determined by the bare VF G_0. Therefore, the equality $\sigma^t_{\rho,N} \approx \sigma^t_{\pi,N}$ in this case can be expected.

C. The cross section for rescattering by a nucleon of a coherently produced system $(p\pi^+\pi^-)$ had a weaker dependence on the mass of the system (Fig. 4).[8]

Interpretation. In our approach the smallness for cross section of rescattering of the $(p\pi^+\pi^-)$ system by a nucleon is attributed to the smallness of the cross section for rescattering of the $p\pi^+$ pair for the resonance mass $M_{p\pi^+} \approx M_{\Delta(1236)}$. The resonance $\Delta^{++}(1236)$ does not occur near the thresold of the $(p\pi^+\pi^-)$ system, and hence, the observed rescattering cross section should have a "normal," large value.

Thus, in terms of the "traditional" diagram approach, we explained qualitatively the basic rules that govern the cross sections for scattering of resonances and particle beams by nucleons. We note that the alternative approach that examines the production of young particles by a nucleus[9] apparently is incapable to explain simultaneously all the rules.

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1) The special case of interaction between the resonance and real photon was examined earlier.[1] It was shown that the multipole momenta of unstable particles (magnetic moment, etc.), in contrast to the stable particles, are complex. This complexity is clearly observed in the experiment. Taking it into account has led to the proper description of the experimental data for radiative scattering $\pi^+ p \to \pi^+ p\gamma$ in the region of the isobar $\Delta^{++}(1236)$.

2) Using the successful description of the processes of interaction between hadrons and soft photons (the Low approximation) and Pomerons[2] in terms of the pole approximation, we expect the amplitudes corresponding to the pole diagrams 1c and 1d with the physical values of VF λ and g_1 as $k^2 \to 0$ can be used to describe the experimental data.

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