Resonance and cyclotron waves on surface electrons

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A surface-electron resonance effect due to anomalous penetration of an electromagnetic field into a metal with specular boundary is predicted. Weakly excited waves of the cyclotron type should exist in the vicinities of the resonance. The conditions for observation are indicated and the shape of the resonance and the wave spectrum are obtained.

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1. If a mirror-surface metal situated in a magnetic field **H** parallel to the boundary, the initial skin layer δ_0 is formed by the glancing electrons (I in

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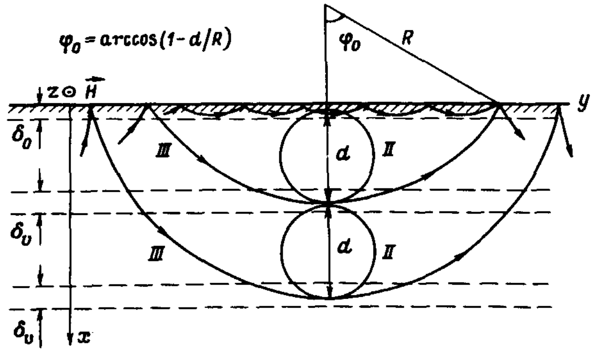

FIG. 1.

Fig. 1). The volume electrons, which enter repeatedly into the skin layer δ_0 (II), transport the electromagnetic field into the interior of the metal and produce current spikes ("skin layers" δ_v) at distances that are multiples of the extremal diameter of the electron orbit $d \gg \delta_v > \delta_0$.[1] Another group of surface electrons (III) falls periodically, with frequency $\pi\Omega/\phi_0$, into the spike, inside of which it moves parallel to the interface (Ω is the cyclotron frequency). The interaction of these electrons with the field of the spike has a resonant character. Resonance takes place when the frequency ω of the external wave is a multiple of the frequency of the periodic motion of the surface electrons

$$\omega = \pi n \Omega / \phi_0 \qquad (n = 1, 2, 3 \ldots). \tag{1}$$

In the vicinity of the resonance (1), the amplitude of the spike is radically changed. This resonance is analogous to the usual cyclotron resonance on volume electrons,[2] and we shall call it cyclotron resonance on surface electrons (CRSE).

For CRSE to occur it is necessary, first, that two different extremal diameters be present ($2R > d$). In addition, the reflection of the resonant electrons from the surface should be close to specular. Finally, it is necessary to satisfy the condition $\nu/\Omega \ll (\pi - \phi_0)/\pi$ in order to distinguish between CRSE and ordinary resonance[2] (ν is the frequency of the electron collisions with the volume scatterers). For simplicity, the Fermi surface is assumed to be axially symmetrical (with the axis along **H**) and the cyclotron frequency is assumed to be the same for all electrons.

We present the results of an asymptotically correct calculation of the distribution of the y component of the field $E(x)$ at high frequencies $\omega \gg \nu$ in the region of the first spike $x = d$:

$$E(x) = E'(0)\frac{\delta_0^2 d}{8a\delta_v}\left(\frac{\delta_0}{\pi|d''|}\right)^{1/2} \frac{\pi\gamma}{\operatorname{sh}(\pi\gamma)}\,\Psi\left(\frac{x-d}{\delta_v}\right). \tag{2}$$

We have introduced here the notation

$$\delta_0 = (4\pi\sqrt{\pi}\, b m e^2 \Omega^2/c^2 h^3)^{-2/5}, \qquad \delta_v = (16\pi a m e^2 \Omega^2/c^2 h^3)^{-1/3},$$

$$a = \int dp_z\, R_\perp(p_z), \qquad b = \int dp_z\, R_\perp^{3/2}(p_z), \qquad \gamma = (\nu - i\omega)/\Omega,$$

e is the absolute value of the charge, m is the cyclotron mass, p_z is the momentum projection, $R_\perp$ is the cyclotron radius of the electron, the integration with respect to p_z is over the Fermi surface and includes summation over all groups, d'' is the second derivative of d with respect to p_z at the extremum point of $d(p_z)$; $E'(0) = dE(x)/dx$ at $x = 0$.

The field distribution function $\Psi(t)$ contains the resonance (1). It has a rather complicated form. At the center of the spike at $t=0$ $(x=d)$ we have

$$\Psi(0) = 1.21\,a^{-2/3}[\,\Theta(-d'') - 0.84\,a^{1/3}\frac{\delta_o}{\delta_v}\,\Theta(d'')\,], \tag{3}$$

where $\Theta(x)$ is the unit step function. Inside the spike and on its wings at $|t| > \delta_0/\delta_v (|x-d| > \delta_0)$ the function $\Psi(t)$ is given by

$$\Psi(t) = \int_0^\infty d\xi\,\frac{\cos[\,t\,\xi - \pi\Theta(d'')/2]}{\xi^3 + a}\,. \tag{4}$$

The quantity α in formulas (3)—(4) is determined by the integral

$$a = \frac{\pi\gamma}{2a}\int dp_z\,R_\perp(p_z)\{\,[1+\Theta(d-2R_\perp)]\,\mathrm{cth}(\pi\gamma) + \Theta(2R_\perp - d)\,\mathrm{cth}[\,\gamma\phi(p_z)]\}\,, \tag{5}$$

where $\phi(p_z) = \arccos(1 - d/R_\perp)$ is the running glancing angle of the glancing electrons (III). It follows from (5) that the CRSE is ensured by those surface electrons whose orbit diameter $2R_\perp(p_z)$, and consequently $\phi(p_z)$, is extremal: $R_\perp = R$, $\phi = \phi_0$. The behavior of the quantity α in the vicinity of the resonance (1) is described by the formula

$$a = -\frac{\pi^2 R}{\sqrt{2}\,a}\,\frac{\mathrm{sgn}\,R''}{(\phi_o|\phi_o''|)^{1/2}}\left(\frac{\omega}{\Delta - i\nu}\right)^{1/2}. \tag{6}$$

The detuning from resonance is $\Delta = \pi n\Omega/\phi_0 - \omega$, and the square root is taken such such that its real part is positive. We note that an analogous resonance should occur also in the next spike near $x = 2d$.

This new resonance effect is manifest in all the high-frequency characteristics of metals, even though it is connected with the anomalous penetration of the field into the sample.

2. The collective motion of the surface electrons leads not only to the resonance (1), but also to the existence of weakly damped magnetic-field oscillations—surface-electron cyclotron waves (SECW). The dispersion equation that determines the spectrum and damping of these waves is obtained by equating the denominator of the integrand in (4) to zero. It is seen from (6) that the solution of this dispersion equation corresponds to the wave at positive values of Δ and R'', and the oscillations themselves are weakly damped at $\Delta \gg \nu$. Since the wave number is $k = \xi/\delta_v$, we obtain for the dispersion and the damping

$$\omega(k) = \frac{\pi n\Omega}{\phi_o}\left[1 - \frac{\pi^4 R^2}{2a^2\phi_o|\phi_o''|}\,(k\delta_v)^{-6}\right] - i\nu\,. \tag{7}$$

The resonant frequencies (1) are the end-point frequencies of the SECW spectra as $k \to \infty$. At small k these waves have no end point of the spectrum. The reason is that the SECW are in fact not natural but forced collective oscillations. Even though they are due to surface electrons, the SECW are obviously not surface oscillations: they exist only in the presence of an external wave incident on the metal. The difference between SECW and natural oscillations manifests itself in the additional condition $\Lambda \ll d$ which is necessary for them to appear ($\Lambda = 6\Delta/k\nu$ is the SECW damping length). Consequently there are no SECW in the collisionless limit—in contrast to the natural oscillations. The inequality $\Lambda \ll d$, which follows from the exact analysis, ensures the dominant role of the surface electrons in the entire wave-localization region $|x-d| \lesssim \Lambda$.

3. The resonance (1) can be observed in the usual manner, by measuring the surface impedance of a bulky sample with specular boundary as a function of the magnetic field. The resonance in the impedance is due to the transport of the spike field into the main skin layer by the electrons of group II. The spectrum of the wave (7) and of the SECW can be observed in a plate by measuring the phase and amplitude of the spike brought out to the boundary by a magnetic field—as the frequency is varied. The value of the resonant effect is larger by $(d/\delta_\nu)^{1/2}$ times than in a bulky sample. The dispersion in the SECW spectrum (7) can also be determined by investigating the amplitude of the transverse sound excited by this wave.

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