Applicability of scale-invariance hypothesis to one-dimensional problems

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A new and better founded formulation is presented of the scale-invariance hypothesis for the calculation of the longitudinal conductivity of a quasi-one-dimensional metal. Results that differ from the those previously obtained by the authors (Sov. Phys. JETP **45**, 118 (1977): Sov. Phys. Solid State **19**, 33 (1977) JETP Lett. **24**, 433 (1976) are presented for a number of quasi-one-dimensional problems. The advanced hypothesis can be directly verified in high-frequency experiments.

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We have previously[1–4] constructed a method of calculating the conductivity of quasi-one-dimensional metallic system, and subsequently applied it to concrete systems such as a quasi-one-dimensional metal with allowance for hops between filaments, to a semimetal with extremely strong magnetic field, and to a quasi-one-dimensional metal with magnetic impurities.

It is known that in a purely one-dimensional metal with random impurities the electrons are localized. Analysis of quasi-one-dimensional systems shows that in all cases it is possible to introduce a dimensionless "delocalization parameter" γ, which determines the delocalizing influence of the non-one-dimensionality of the system or the inelasticity of the collisions (with phonons or magnetic impurities). If $\gamma \gg 1$, then the kinetic equation is applicable. If $\gamma \ll 1$, then the influence of the localization effect is very strong, and the conductivity is small. It turns out then that the conductivity is not an analytic function of γ as $\gamma \to 0$, and can therefore not be determined by expansion in γ. Since the complete solution of the problem is in most cases complicated, we go around this difficulty in[2–4] with the aid of the similarity hypothesis.

Namely, we took a finite sample of length L and found the first correction to the conductivity in γ. At small γ, the conductivity of such a sample is analytic. We then advanced the hypothesis that in the general case similarity holds true, and the conductivity σ is given by

$$\sigma = Aqf(\gamma/q^{\nu}), \tag{1}$$

where $A=\text{const}$, $q=\exp(-L/4l_2)$, l_2 is the mean free path with scattering $p_0\to -p_0$, $f(0)=1$, $f(x\to\infty)\propto x^{1/\nu}$. It could therefore be concluded that $\sigma\propto\gamma^{1/\nu}$ as $L\to\infty$. The degree ν was determined from the expansion of σ in γ for $q\neq 0$.

This procedure is in fact doubtful, since a limitation of the electron correlations arises in a quasi-one-dimensional objects at $\gamma\neq 0$ for finite scattering, so that the interchange of the limits $L\to\infty$ and $\gamma\to 0$ is not a neutral procedure. Furthermore, a very suspicious circumstance is the fact that none of the expansions in γ obtained in[2-4] are strictly speaking in a scaling form.

In view of the foregoing we shall use a new procedure, which we regard as better founded. We calculate the static conductivity as the limit $\lim_{\omega\to 0}\sigma(\omega)$. For the conductivity $\sigma(\omega)$ we have a formula of the type (see[1,2])

$$\sigma(\omega) = \frac{e^2v^2}{\pi S}\,\mathrm{Sp}\,\sigma\int_{-\infty}^{\infty}[\sigma_3 G_{R\omega}(zz_1)\sigma_3 G_A(z_1 z)]dz_1 . \tag{2}$$

At $\omega\neq 0$ the coefficients of the expansion of σ in γ are finite, i.e, $\sigma(\omega)$ is analytic in γ at small γ. In the zeroth order in γ we get (see[1])

$$\sigma_0 = \frac{8\,\zeta(3)}{\pi S}\,\frac{e^2 l_2^2\omega}{iv} . \tag{3}$$

Here S is the area of the crystal cell in the (xy) plane, l_2 is the free path for backward scattering by the impurities $(p_0\to -p_0)$. The expansion in γ depends on the particular problem, but in all cases only powers of ω appear, i.e., the expansion is of the "scaling" type. Consequently, we propose in lieu of (1)

$$\sigma = \sigma_0 f\left[\gamma\left(\frac{iv}{\omega l_2}\right)^{\nu}\right], \tag{4}$$

where f has the same properties as before.

The results for the longitudinal conductivity, which are obtained in this case, differ from those obtained in[2-4]. We cite them without proof:

a) The longitudinal conductivity of a quasi-one-dimensional metal (cf.[2]). In this case $\nu=1$,

$$\sigma\sim(e^2/\pi S)(\overline{a}^2 l_2^2/v^2)(l_2^{-1}+l_1^{-1})^{-1}. \tag{5}$$

b) The longitudinal conductivity of a semimetal in a very strong magnetic field, for Coulomb impurities (cf.[3]). In this case $\nu=2$,

$$\sigma = A\frac{e^2 l_2}{(2\pi\lambda)^2}\left(\frac{\gamma}{\gamma_0}\right)^{1/2} = \frac{A}{\pi^3}\,\frac{p_0^6\epsilon_0^2}{N_i m^2 Z^2 e^2}\,\ln\left(\frac{1}{2p_0\lambda}\right), \tag{6}$$

i.e., at $n_e=\text{const}$, $p_0\sim n_e\lambda^2\propto H^{-1}$ and $\sigma\propto H^{-6}$.

c) Conductivity of quasi-one-dimensional metal with magnetic impurities $(\nu=2)$, cf.[4]

$$\sigma \sim \frac{e^2}{\pi S} \frac{l_2^2}{l_M}, \tag{7}$$

this means that σ is proportional to the concentration N_m of the magnetic atoms. Formulas (6.7) and (6.8) of[4] for the function $\sigma(H)$ remain in force. We note that relation (6.8) was confirmed by experiment[5] at $\mu H \ll T$.

An additional argument in favor of the new scaling hypothesis is the fact that in model (a) the conductivity could be calculated exactly. The result agrees in order of magnitude with formula (5). The coefficient is equal to $46\zeta(3)$.

Formula (4) can be verified in high-frequency experiments. It can also be written in the form

$$\sigma = \sigma(0)\,\phi\left(\frac{\omega l_2}{iv} / \gamma^{1/\nu}\right), \tag{4'}$$

which is more convenient at low frequencies. Here $\sigma(0) \sim \gamma^{1/\nu}\sigma_0(iv/\omega l_2)$. The coefficients of the expansion of σ in $(-i\omega)$ are finite. It follows therefore that in first-order approximation $\mathrm{Im}\sigma(\omega)$ does not depend on γ and differs from σ_0 in (3) only by a numerical factor. Formula (3) yields $\mathrm{Im}\sigma(\omega)$ at $1 \gg \omega l_2/v \gg \gamma^{1/\nu}$.

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