Metal-insulator transition of an electron-hole liquid in a strong magnetic field

V. S. Babichenko and T. A. Onishchenko

P. N. Lebedev Institute, USSR Academy of Sciences
(Submitted June 4, 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **26**, No. 2, 75–78 (20 July 1977)

A semimetal in a strong magnetic field is considered. It is shown that the ground state of such a system is unstable and that a metal-insulator transition takes place, the restructuring of the ground state being due to plasma instability.

PACS numbers: 71.30.+h, 71.35.+z

Keldysh and Onishchenko[1] have calculated the ground-state energy of a homogeneous electron-hole liquid in a strong magnetic field $H \gg 1$ at densities $\lambda^{-2} \ll n \ll \lambda^{-3}$, where $\lambda \ll 1$ is the magnetic length. They have shown that the ground-state energy has a minimum at $n = n_0 \sim \lambda^{-16/7}$, with $E_{\mathrm{corr}/N} \sim n^{1/4}$ at $\lambda^{-2} \ll n \ll \lambda^{-8/3}$ and $E_{\mathrm{corr}/N} \sim \ln n$ at $\lambda^{-8/3} \ll n \ll \lambda^{-3}$. The transition of such a liquid into the state of an excitonic dielectric was considered in[2–4]. It was shown in[3,4], in the "parquet" approximation, that the spectrum of the elementary excitations has a dielectric gap. The results of these studies are valid in the density region $\lambda^{-8/3} \sim n \ll \lambda^{-3}$. We consider in the present paper the density region $n_0 \lesssim n \ll \lambda^{-8/3}$. We shall show that the homogeneous ground state is unstable and that a density wave with momentum $2p_F$ is produced in the system along the magnetic field, and at the same time a dielectric gap is produced in the elementary-excitation spectrum. The gap equals $\Delta = \epsilon_F/15$ at $n = n_0$ and decreases exponentially with increasing density. If the electron and hole masses are equal, then the amplitude of the electron density wave is equal to the amplitude of the hole density wave, and their phases coincide, and since these waves correspond to opposite charges, there is no charge-density wave in the system. On the other hand if the electron and hole masses are not equal, then the electron and hole density waves are likewise unequal, and a charge-density wave is produced. We note that in our density region the appearance of a dielectric gap in the elementary-excitation spectrum is not due to pairing of electrons and holes from different

 0021-3640/77/2602-0068$00.60

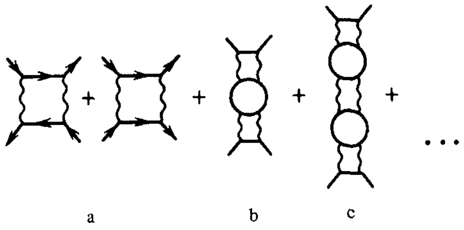

FIG. 1. Sequence of principal diagrams for $\Gamma(p, p', k)$.

bands, as is the case in an excitonic dielectric,[2-5] but is due to plasma instability of the ground state. In a strong magnetic field $H \gg 1$ the transitions between the Landau levels can be neglected, and then the electron dispersion laws become one-dimensional and take the form

$$\epsilon_1(\mathrm{p}) = \frac{p_\parallel^2}{2m_1} - \frac{p_F^2}{2m_1}\ ;\ \epsilon_2(\mathrm{p}) = \frac{p_F^2}{2m_2} - \frac{p_\parallel^2}{2m_2}\ .$$

The results are valid also for electron-hole drops in semiconductors in a strong magnetic field. We shall henceforth use the system of units $e/\sqrt{\epsilon} = \hbar = m_1 = 1$ and assume that $\sigma = m_2/m_1 \geq 1$. We consider the class of diagrams that are principal for the complete two-particle vertex parts $\Gamma_{\alpha\beta}(p, p', k)$, where α, $\beta = 1, 2$ are the numbers of the bands of the interacting electrons, p and p' are the momenta of the incoming electron ends, and k is the transfer momentum. A distinguishing feature of one-dimensional systems and of a system in a strong magnetic field is the presence of several channels, which produce logarithmic divergences. The result is that at $n \gtrsim \lambda^{-8/3}$ the diagrams for $\Gamma_{\alpha\beta}(p, p', k)$ constitute a class of diagrams, called a "parquet," which was considered in[3,4]. It will be shown that at $n_0 \lesssim n \ll \lambda^{-8/3}$ the class of principal diagrams for $\Gamma_{\alpha\beta}(p, p', k)$ is much simpler. We separate in $\Gamma_{\alpha\beta}$ the vertex part that is irreducible in the Coulomb line $V(\mathrm{k}) = 4\pi/|\mathrm{k}|^2$ and designate it $\tilde{\Gamma}_{\alpha\beta}(p, p', k)$. In the density region $n_0 \lesssim n \ll \lambda^{-8/3}$, the principal diagrams for $\tilde{\Gamma}_{\alpha\beta}$ will be those shown in Fig. 1. The interaction lines in these diagrams correspond to the screened Coulomb potential $U = V/[1 - (\Pi_1^{(0)} + \Pi_2^{(0)})V]$, where $\Pi_\alpha^{(0)}$ is a polarization operator, for which an expression is given in[1]. Let us explain why the principal diagrams for $\tilde{\Gamma}_{\alpha\beta}$ will be those in Fig. 1. Elaboration of the diagrams $\gamma_{\alpha\beta}$ of Fig. 1(a) via "scattering" channels (diagrams of the ladder type) give rise to a factor $p_F^{-1}\ln(\epsilon_F/\Delta)$. We shall show that this factor is small in the density region considered by us, by using for the gap the final equation $\Delta = 8\epsilon_F \exp\{(8\pi^2\lambda^2 p_F/\gamma)\}$, where $\gamma \sim -n^{-3/4}$.[6] We obtain

$$p_F^{-1}\ln\frac{\epsilon_F}{\Delta} \sim p_F^{-1}\frac{\lambda^2 p_F}{\gamma} \sim n^{3/4}\lambda^2 << 1$$

at $n \ll \lambda^{-8/3}$.

On the other hand, elaboration of the diagrams for $\gamma_{\alpha\beta}$ via the "annihilation" electron-hole channel with a momentum transfer $k_\parallel = 2p_F$ (Figs. 1b, 1c) yields a factor on the order of unity $\gamma\Pi^{(0)}(\Delta, 2p_F) \sim \gamma(\lambda^2 p_F)^{-1}\ln(\epsilon_F/\Delta) \sim \gamma(\lambda^2 p_F)^{-1} \times (\lambda^2 p_F/\gamma) \sim 1$. This makes it necessary to take into account for $\Gamma_{\alpha\beta}$ the entire sequence of diagrams of the type shown in Fig. 1. It is this sequence of diagrams which is the principal one for $\tilde{\Gamma}_{\alpha\beta}$. It is important that $\gamma_{\alpha\beta}(p, p', k)$ does not depend on $\mathbf{p}_\perp$ or $\mathbf{p}'_\perp$, since the single-particle Green's function $G(p)$ does not depend on $\mathbf{p}_\perp$. It is easily seen that the main contribution to the integral with respect to $q=(\omega, q_\parallel, \mathbf{q}_\perp)$ for $\gamma_{\alpha\beta}(p,p',k)$ (Fig. 1a) is made by $\omega \sim n^{1/2}$; $q_\parallel \sim |\mathbf{q}_\perp| \sim n^{1/4} \gg p_F$ and therefore $\gamma_{\alpha\beta}(p,p',k)$ can be regarded as a constant if $p_\parallel$, $p'_\parallel$, $|\mathbf{k}| \ll n^{1/4}$ and the frequency components of these momenta are much smaller than $n^{1/2}$. Assuming $\gamma_{\alpha\beta}$ to be constant, we obtain a system of two algebraic equations for $\Gamma_{\alpha\beta}(k)$

$$\Gamma_{\alpha\beta}(k) = (V(\mathbf{k}) + \gamma_{\alpha\beta}) + \sum_{\beta'=1}^{2} (V(\mathbf{k}) + \gamma_{\alpha\beta'})\Pi^{(o)}_{\beta'}(k)\Gamma_{\beta'\beta}(k), \qquad (1)$$

where

$$\gamma_{\alpha\beta} = -\frac{(-1)^{\alpha+\beta}}{2n^2}\int \frac{d^3\mathbf{q}\,d\omega}{(2\pi)^4}\left[\frac{V(\mathbf{q})}{1-(\Pi^{(o)}_1(\omega,q)+\Pi^{(o)}_2(\omega,q))V(\mathbf{q})}\right]^2$$

$$\times\ \Pi^{(o)}_\alpha(\omega,q)\,\Pi^{(o)}_\beta(\omega,q). \qquad (2)$$

Solution of the system (1) yields:

$$\Gamma_{11} = [V(1-\Pi^{(o)}_2\gamma) - \Pi^{(o)}_2(\gamma_{11}\gamma_{22}-\gamma_{12}^2) + \gamma_{11}]D^{-1}$$

$$\Gamma_{12} = (V+\gamma_{12})D^{-1} \qquad (3)$$

$$\Gamma_{22} = [V(1-\Pi^{(o)}_1\gamma) - \Pi^{(o)}_1(\gamma_{11}\gamma_{22}-\gamma_{12}^2) + \gamma_{22}]D^{-1},$$

where

$$\gamma = \gamma_{11}+\gamma_{22}-2\gamma_{12}$$

$$D = 1 - V(\Pi^{(o)}_1+\Pi^{(o)}_2)\left(1-\gamma\frac{\Pi^{(o)}_1\Pi^{(o)}_2}{\Pi^{(o)}_1+\Pi^{(o)}_2}\right) + \Pi^{(o)}_1\Pi^{(o)}_2(\gamma_{11}\gamma_{22}-\gamma_{12}^2)$$

$$-(\gamma_{11}\Pi^{(o)}_1+\gamma_{22}\Pi^{(o)}_2).$$

Calculating the integrals (2) for $\gamma_{\alpha\beta}$ we find that

$$\gamma = \gamma_{11}+\gamma_{22}-2\gamma_{12} = -n^{-3/4}\,2^{9/4}\,\pi^{3/4}\,[\Gamma(1/4)]^{-2}f(\sigma) \qquad (4)$$

$$f(1) = 1$$

For the case of equal masses $\sigma=1$ we obtain $\Pi^{(0)}_1=\Pi^{(0)}_2=\Pi$ $\gamma_{11}=\gamma_{22}=-\gamma_{12}=\gamma/4$, and the expressions for the vertices simplify to

$$\Gamma_{11}=\Gamma_{22}=\frac{V(1-\Pi\gamma)+\frac{1}{4}\gamma}{(1-\frac{1}{2}\gamma\Pi)(1-2V\Pi)},\quad \Gamma_{12}=\frac{V+\frac{1}{4}\gamma}{(1-\frac{1}{2}\gamma\Pi)(1-2V\Pi)}. \qquad (5)$$

The poles in the vertices $\Gamma_{\alpha\beta}(k)$ are determined by the equation $D=0$. For the case of equal masses, the pole of the vertex parts $\Gamma_{\alpha\beta}(k)$ corresponds, as a result of the vanishing of the expression $1-\frac{1}{2}\gamma\Pi(k)=0$, where $k=(\Omega, k_{\parallel}=2p_F, k_{\perp}=0)$, to instability of the ground state and to the formation in the system of a density wave with momentum $k_{\parallel}=2p_F$.

The expression for the pole is

$$\Omega_0 = 8\epsilon_F \exp\left\{-\frac{8\pi^2\lambda^2 p_F}{|\gamma|}\right\} = 8\epsilon_F \exp\left\{-\frac{25}{4}\left(\frac{n}{n_0}\right)^{7/4}\right\}. \tag{6}$$

At $n=n_0$ we get $\Omega_0=\epsilon_F/15$. The restructed state can be obtained by writing down the Gor'kov equations, which is done in standard fashion. The solution of these equations introduces into the spectrum of the elementary excitations of the restructured state a dielectric gap Δ equal in magnitude to $\Delta=\Omega_0$. We note that at $n=n_0$ we have $\Delta=\epsilon_F/15$ and we have only a numerical smallness of the gap in comparison with the Fermi energy, but this numerical smallness makes it possible to reconstruct the state in standard fashion without any additional complications. Results similar to those obtained here are valid also for quasi-one-dimensional systems, for which the ground-state energy was calculated in[6]. We note that the electron gas on the positive compensating background, has no instability, in contrast to the electron-hole liquid considered in the present paper.

In conclusion, the authors thank L. V. Keldysh for numerous useful discussions.

[1] L. V. Keldysh and T. A. Onishchenko, Pis'ma Zh. Eksp. Teor. Fiz. 24, 70 (1976) [JETP Lett. 24, 59 (1976)].
[2] E. W. Fenton, Phys. Rev. 170, 816 (1968).
[3] A. A. Abrikosov, J. Low Temp. Phys. 2, 37 (1970); 10, 3 (1973).
[4] S. A. Brazovskiĭ, Zh. Eksp. Teor. Fiz. 62, 820 (1972) [Sov. Phys. JETP 35, 433 (1972)].
[5] L. V. Keldysh and Yu. V. Kopaev, Fiz. Tverd. Tela 6, 2791 (1964) [Sov. Phys. Solid State 6, 2219 (1965)].
[6] E. A. Andryushin, V. S. Babichenko, L. V. Keldysh, T. A. Onishchenko, and A. P. Silin, Pis'ma Zh. Eksp. Teor. Fiz. 24, 210 (1976) [JETP Lett. 24, 185 (1976)].