Spin diffusion in solid He^3

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(Submitted June 13, 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **26**, No. 2, 86–89 (20 July 1977)

It is shown that the presence of a macroscopic region with fully polarized nuclear spins around a vacancy in solid He^3 leads to an increase and a peculiar temperature dependence of the spin-diffusion coefficient, which decreases under certain conditions with increasing temperature.

PACS numbers: 67.80.Mg

Andreev[1] has shown that at temperatures $I < T < \Delta$ ($I \sim 10^{-3}$ K is the value of the exchange integral, T is the temperature, and $\Delta \sim 10$ K is the width of the energy gap) favorable energy conditions are created for the onset of a macroscopic ordered region (OR), in which the nuclear spins are fully polarized. The presence of such OR should exert a substantial effect on the magnetic properties of He^3 crystals.

Spin diffusion processes in He^3 crystals have been under intensive investigation recently,[2] and it is therefore of interest to examine the effects of OR on these processes.

 0021-3640/77/2602-0078$00.60

We shall analyze spin diffusion processes within the framework of the following simple model: We replace the system of ordered nuclear spins around the vacancy by a single spin of value $S=N_0 s$ (where N_0 is the number of nuclei in the OR, $N_0 \gg 1$, and s is the nuclear spin) and placed at the center of a spherical OR of radius R (we shall henceforth assume that the temperature is low enough to be able to disregard the motion of the vacancies). We represent the exchange-interaction Hamiltonian in the form of two parts: the first is the interaction between the disordered spin with exchange integral I (we assume that only the nearest neighbors interact); the second part is the interaction between the ordered and disordered spins, and the fact that only the ordered spins situated on the OR surface take place in the interaction will be taken into account by renormalizing the exchange integral, which in this case is equal to a/R (where a is the period of the lattice).

With the chosen model taken into account, the Hamiltonian of the problem is

$$H = H_z + H_{\text{exc}}$$

$$H = \omega_I \sum_i s_i^z + \omega_I \sum_n S_n^z \tag{1}$$

$$H = \sum_{ij} I_{ij}(\mathbf{s}_i \mathbf{s}_j) + \sum_{in} I_{in}(\mathbf{s}_i \mathbf{S}_n),$$

where $\omega_I \Sigma_i s_i^z$ is the Zeeman energy of the disordered spins, $\omega_I \Sigma_n S_n^z$ is the Zeeman energy of the ordered spins (the summation is over the number of the OR), $\Sigma_{ij} I_{ij}(s_i s_j)$ is the exchange interaction between the ordered spins, and $\Sigma_{in} I_{in}(s_i S_n)$ is the exchange interaction between the disordered and ordered spins.

We introduced the spatial density of the Zeeman Hamiltonian, such that: $\int H_z(x)\,dx = H_z$

$$H_z(x) = \omega_I \sum_i \delta(x - x_i) s_i^z + \omega_I \sum_n \delta(x - x_n) S_n^z .$$

The equation of motion for the density of the Zeeman Hamiltonian is

$$\frac{dH_z(x)}{dt} = \frac{1}{i}[H_z(x), H_{\text{exc}}] = K_z(x). \tag{2}$$

On the basis of (2) we can construct the Zubarev nonequilibrium statistical operator,[3] which takes in the high-temperature approximation the form

$$\rho \approx \frac{1}{\text{Sp}\,1}\left(1 - \int H_z(x)\beta_z(x)dx + \int_{-\infty}^{0} e^{\epsilon t}\,dt \int dx'\, K_z(x't)\beta_z(x')\right), \tag{3}$$

where

$$A(t) = \exp[iHt]\, A \exp[-iHt].$$

Averaging (2) with the aid of (3) and assuming that the diffusion approximation

$$\beta_z(x') = \beta_z(x) + (x'_\alpha - x_\alpha)\frac{\partial \beta_z}{\partial x_\alpha} + \frac{(x'_\alpha - x_\alpha)(x'_\beta - x_\beta)}{2}\frac{\partial^2 \beta_z}{\partial x_\alpha \partial x_\beta}$$

is valid, we obtain

$$D_z^{\alpha\beta} = -\frac{1}{\langle H_z(x) H_z\rangle} \int dx' \frac{(x'_\alpha - x_\alpha)(x'_\beta - x_\beta)}{2} \int_{-\infty}^{0} e^{\epsilon t} dt \langle K_z(x) K_z(x' t)\rangle,$$

where

$$\langle \dots \rangle = \frac{\mathrm{Sp}\dots}{\mathrm{Sp}1}.$$

Without presenting the calculation and assuming that only the nearest neighbors interact, we get

$$D = D_0 \frac{1 + \frac{\sqrt{2}}{2a}\mu N^{5/3}}{(1+\mu N_0)\left(1 + \frac{1}{4a}\mu N_0\right)^{1/2}}, \tag{4}$$

where μ is the relative concentration of the ordered spins ($\mu < 1$). D_0 is the coefficient of spin diffusion in an ideal crystal lattice,[4] α is the number of the nearest neighbors of the He^3 nuclei. N_0 and μ depend on the temperature in the following manner[1]

$$N_0 = AT^{-3/5}, \qquad \mu = AT^{-3/5}\exp\left\{-\frac{\epsilon_0}{T} - \frac{B}{T^{3/5}}\right\},$$

$$A = \frac{4}{3}\pi n\left(\frac{\pi}{4\ln 2}\frac{\hbar^2}{Mn}\right)^{3/5}, \qquad B = \frac{5}{6}\left(\frac{\pi^2\hbar^2}{M}\right)^{3/5}(4\pi n \ln 2)^{2/5}.$$

M is the effective mass and ϵ_0 is the minimal energy of the vacancy.

1. We consider the case $\mu N_0 \ll 1$. Assuming that $(\sqrt{2}/2a)N_0^{2/3} > 1$, we obtain

$$D \approx D_0\left[1 + \frac{\sqrt{2}}{2a}\mu N_0^{5/3}\right]$$

i.e., the diffusion coefficient in a crystal with vacancies is larger than D_0 and has a temperature dependence given by

$$D \approx D_0\left[1 + \frac{\sqrt{2}}{2a}A^{8/5}\exp\left\{-\frac{\epsilon_0}{T} - \frac{B}{T^{3/5}}\right\}\frac{1}{T^{8/5}}\right], \tag{5}$$

and increases with temperature because of the strong exponential dependence.

It is known that the exponential growth of D with temperature is the consequence of the hopping mechanism of the diffusion—the hopping of the vacancies[5] (which we do not take into account). The growth of D with temperature obtained by us [Eq. (5)] differs in form from the hopping mechanism of the diffusion.

2. We consider now the case $\mu N_0 \gg 1$. In this case

$$D \approx D_0 \sqrt{\frac{2}{a}} \frac{N_0^{1/6}}{\mu^{1/2}},$$

and obviously $D \gg D_0$. The temperature dependence takes the form

$$D \approx D_0 \sqrt{\frac{2}{a}} A^{-1/3} T^{1/5} \exp\left\{\frac{\epsilon_0}{2T} + \frac{B}{2T^{3/5}}\right\}. \tag{6}$$

We see that the obtained temperature dependence is substantially different, and D decreases with increasing temperature, in contrast to the preceding case.

In conclusion, the authors thank A. F. Andreev for interest in the work.

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