Mechanism for the radio emission of pulsars

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An instability of the flow of a relativistic electron-positron plasma in a strong curvilinear magnetic field is predicted theoretically. This instability can explain the origin and basic properties of the observed radio emission of pulsars.

Soon after the 1967 discovery of the pulsating radio sources now known as pulsars, they were identified as rotating neutron stars.[1] More than 400 pulsars have now been identified. The general properties of their magnetosphere have been determined, as have the properties of the electron-positron plasma that flows in it.[1,2] It has been

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found that the observed radio frequencies ω are close to the frequencies of the so-called bending radiation $\omega_c = c/p\gamma^3$ which is generated by particles moving along curved magnetic lines of force of radius ρ with a characteristic energy $\gamma = \epsilon/m_0c^2 \sim 200 - 500$. However, the actual mechanism for the generation of this radio emission, which is undoubtedly a collective plasma effect, has remained undetermined.[1,2]

The fundamental problem of the electrodynamics of a relativistic plasma moving in a strong curvilinear magnetic field has yet to be formulated and solved; in fact, there is no linear theory describing the growth of perturbations in such a plasma. In the present letter we offer a solution of the general problem, and we show that the application of the results of this theory of the magnetosphere of pulsars can explain the origin and basic properties of the observed radio emission.

To study the electrodynamic properties of a steady-state plasma flow, we need to find the dielectric tensor $\epsilon_{\alpha\beta}(\omega,\mathbf{k},\mathbf{r})$. The general expression for $\epsilon_{\alpha\beta}(\omega,\mathbf{k},\mathbf{r})$ in a collisionless, inhomogeneous plasma is

$$\epsilon_{\alpha\beta} = \delta_{\alpha\beta} - \frac{4\pi i}{\omega} e^2 \int d\mathbf{p} v_\alpha$$

$$\int_{-\infty}^{t} dt' \exp[i\omega(t-t') - i\mathbf{k}\vec{\eta}^*] \det{}^{-1} \left| \delta_{\mu\nu} - \frac{\partial\lambda_\mu\left(\mathbf{r}+\frac{\eta^*}{2}\right)}{\partial r_\nu} \right| \cdot$$

$$\cdot \left[\left(1 - \frac{\mathbf{k}\mathbf{v}'}{\omega}\right)\delta_{\beta\sigma} + \frac{k_\sigma v'_\beta}{\omega} + \frac{i}{2\omega}\frac{\partial}{\partial r_\sigma} v'_\beta - \frac{i}{2\omega}\delta_{\beta\sigma}\frac{\partial}{\partial r_\chi} v'_\chi\right] \frac{\partial F\left(\mathbf{r}+\frac{\vec{\eta}^*}{2}, \mathbf{p}\right)}{\partial p'_\sigma} . \quad (1)$$

Here $F(\mathbf{r},\mathbf{p})$ is the unperturbed distribution function of the particles, of charge e; and $\mathbf{p}' = \mathbf{p}(t')$, $\mathbf{v}' = \mathbf{v}(t')$, and $\mathbf{r}' = \mathbf{r}(t')$ are the momentum, velocity, and coordinates at the time t' of a particle which is moving along an unperturbed trajectory, so that at the time t this particle has a momentum $\mathbf{p}$, a velocity $\mathbf{v}$, and a coordinate $\mathbf{r}$. The vector $\vec{\eta}^*(\mathbf{r},\mathbf{p},t-'t)$ is the solution of the equation

$$\vec{\eta}^* = \vec{\lambda}\left(\mathbf{r} + \frac{\vec{\eta}^*}{2},\ \mathbf{p}, t-t'\right),$$

where the function $\vec{\lambda}$ describes the trajectory of the particle, $\mathbf{r} = \mathbf{r}' + \vec{\lambda}(\mathbf{r},\mathbf{p},t-t')$.

The dielectric constant in (1) completely determines the dispersion properties of the inhomogeneous media if the wavelength is small in comparison with the length scale of the inhomogeneity (which in our case is the radius of curvature, ρ) and if the wave damping (or growth) is relatively slow:

$$k\rho \gg 1, \quad |\vec{\kappa}|/|\mathbf{k}| \ll 1, \quad \kappa = \operatorname{Im} k. \quad (2)$$

The electric field of the wave is then described by the geometric-optics equations, and Maxwell's equations reduce at each point $\mathbf{r}$ to an ordinary dispersion relation with the tensor $\epsilon_{\alpha\beta}$, in (1).

Now using general expression (1), we find the following expression for an electron-positron plasma which is moving at a relativistic velocity $v_{\parallel}$ along a very strong $(\omega \ll \omega_B = eB/m_0 c\gamma)$ curvilinear magnetic field:

$$\epsilon_{\alpha\beta} = \begin{pmatrix} 1 + 4\pi\omega_p^2 \frac{\rho^{2/3}}{k_z^{4/3}} \left\langle \frac{[\mathrm{Gi}'''(\zeta) - i\mathrm{Ai}'''(\zeta)]}{\gamma^3 v_{\parallel}^2} \right\rangle & 0 & -4\pi\omega_p^2 \frac{\rho}{k_z} \left\langle \frac{[\mathrm{Ai}''(\zeta) + i\mathrm{Gi}''(\zeta)]}{\gamma^3 v_{\parallel}^2} \right\rangle \\ 0 & 1 & 0 \\ 4\pi\omega_p^2 \frac{\rho}{k_z} \left\langle \frac{[\mathrm{Ai}''(\zeta) + i\mathrm{Gi}''(\zeta)]}{\gamma^3 v_{\parallel}^2} \right\rangle & 0 & 1 + 4\pi\omega_p^2 \frac{\rho^{4/3}}{k_z^{2/3}} \left\langle \frac{[\mathrm{Gi}'(\zeta) - i\mathrm{Ai}'(\zeta)]}{\gamma^3 v_{\parallel}^2} \right\rangle \end{pmatrix} \tag{3}$$

Here $\omega_p^2 = 4\pi e^2 N/m_0$ is the plasma frequency, N is the plasma density, γ is the Lorentz factor of the particle, and the Airy functions are defined by

$$\mathrm{Ai}(\zeta) + i\mathrm{Gi}(\zeta) = \frac{1}{\pi} \int_0^\infty \exp\left(i\zeta\tau + i\frac{\tau^3}{3}\right) d\tau .$$

The primes in (3) mean derivatives with respect to the argument

$$\zeta = 2(\omega - k_z v_{\parallel}) \frac{\rho^{2/3}}{k_z^{1/3} v_{\parallel}} .$$

The coordinate system has been chosen in such a way that the z axis runs along the magnetic field, while the x axis is orthogonal to z in the plane of the magnetic field. The angle brackets mean an average over the distribution function of the particles: $\langle \psi \rangle = \int dp_{\parallel} f(p_{\parallel}) \psi$.

Using the expression found for $\epsilon_{\alpha\beta}$, we can determine the normal waves. Their properties depend strongly on the parameter

$$a = 4\pi \left\langle \frac{1}{\gamma^3} \right\rangle \omega_p^2 \frac{\rho^{4/3}}{k_z^{2/3} c^2} . \tag{4}$$

If $a \gg 1$ and $(\omega p/c\gamma^3)^{2/3} \gg a$ (region II in Fig. 1), two longitudinal waves and two transverse waves in the rf range can propagate through the magnetospheric plasma, as in the case of a straight magnetic field. If the conditions $a \gg 1$ and $(\omega p/c\gamma^3)^{2/3} \ll a$ hold (region I), on the other hand, one of the plasma waves splits in three. Two of these three waves are unstable at small angles $\theta \lesssim a(k_z\rho)^{-1/3}$ (θ is the angle between **k** and **B**). Normal modes of this type correspond to drift waves which are propagating along the magnetic field. An instability occurs only if the number density of particles is high ($a \gg 1$), and it is of a hydrodynamic nature: $\mathrm{Im}k \propto N^{1/5}$.

Under the conditions prevailing in the magnetosphere of a pulsar (with a dipole magnetic field, $N \propto r^{-3}$, where r is the distance from the center of the star), region I, in which rapidly growing waves propagate, is close to the neutron star, at $r \leqslant 10^2$–10^3 km. In region III ($a \ll 1$), which is more remote, there is no instability, and only two

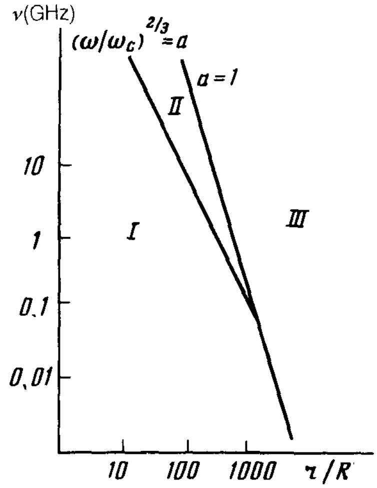

FIG. 1. Three different regions of parameter values in the magnetosphere of a pulsar ($\gamma_0 = 300$). Here R is the radius of the star.

transverse waves, with $n \equiv kc/\omega \simeq 1$, can propagate. The conversion of the unstable plasma modes into a transverse wave occurs at the parameter value $a \simeq 1$.

The total optical thickness traversed by the unstable normal modes as they propagate from the surface of the neutron star out to the conversion point, $a \simeq 1$, is found to be

$$\tau_{1,2} = 2\frac{\omega}{c}\int \mathrm{Im}\, n\, dl = -290 s_{1,2}\, \nu_{\mathrm{GHz}}^{1/5} \left(\frac{\gamma_0}{300}\right)^{-3/5} . \qquad (5)$$

Here $s_{1,2}$ is a geometric factor on the order of unity, ν_{GHz} is the wave frequency in gigahertz, and $\gamma_0 \simeq 200-500$ is a characteristic value of the Lorentz factor of the particles moving in the magnetosphere of the pulsar.[1]

We see that the amplification is huge over the entire range of observed frequencies (30 MHz to 10 GHz). Consequently, the radiated power will essentially always be limited by nonlinear processes, so that it may constitute a definite fraction of the total energy of the plasma that generates it, as is confirmed by observational data.[3] The conversion region, with $a \simeq 1$, determines the directionality of the emission. Also agreeing completely with observations is the dependence of the width (W) of the directional pattern on the frequency of the radio emission: Observations yield[4] $W \propto \nu^{-\alpha}$ with $\alpha = 0.16 \pm 0.05$, while the theory predicts $\alpha = 1/7$. Furthermore, the width W itself corresponds well to the observational data.

It can thus be concluded that the theory outlined here explains the origin and basic properties of the observed radio emission of pulsars.

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