Quasilinear stabilization of ballooning modes in toroidal plasma systems

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An equation describing the quasilinear saturation of ballooning modes in toroidal configurations is derived. The ballooning instabilities are effectively stopped because the pressure profile near resonant surfaces becomes flattened as a result of the formation of magnetic islands. There is no fundamental change in the structure of the overall magnetic configuration.

The ballooning instabilities of a toroidal plasma are presently regarded as the basic limitation on the pressure in closed magnetic-confinement systems, in particular, a tokamak reactor. The manifestations of this limitation in real plasmas are now being studied extensively. For the simple model of a resistive plasma, for example, it has been shown[1] that near the threshold in terms of ideal modes the ballooning instabilities

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appear as reconnection modes in the linear stage.[2] By analogy with the classical $m = 1$ reconnection mode,[3] for which there is no nonlinear saturation, one might expect that in their nonlinear stage the ballooning instabilities would also change the structure of the entire region affected by the linear perturbation.

In this letter we show that the quasilinear flattening of the pressure profile upon the formation of a system of magnetic islands causes a ballooning mode to reach saturation while still in an early stage, without leading to a fundamental change in the configuration. We calculate the saturation amplitude.

To simplify the calculations, we consider the asymptotic case of ballooning modes with large wave numbers, m, $n \gg 1$ (the Connor-Hastie-Taylor theory[4]). In this case we can make use of the equivalence of the harmonics of a perturbation. Examining a single mode $n = \text{const}$, we may assume that after the transients have died out—after the decay of the δ-function currents near the resonant surfaces—a system of magnetic islands forms on each of these surfaces. The pressures (p) profile, governed by the condition $\mathbf{B}\nabla p = 0$, will also acquire an island structure, so that there will be an effective flattening of the p profile near resonances. The steady-state amplitude of a perturbation can be estimated by assuming that the quasilinear calculation of a near-equilibrium situation is equivalent to the problem of the neutral stability of the original configuration with a modified pressure profile, flattened over a width of $2w$ near resonant surfaces, as shown in Fig. 1.

To illustrate the quasilinear saturation, we consider a model tokamak with the simple geometry $g_{11} = 1$, $g_{12} = 0$, $g_{22} = a^2$, $g_{33} = (R - a\cos\theta)^2$ (R and a are the major radius and instantaneous minor radius of the torus). We recall that in the theory of ballooning modes there are two length scales: the distance between adjacent

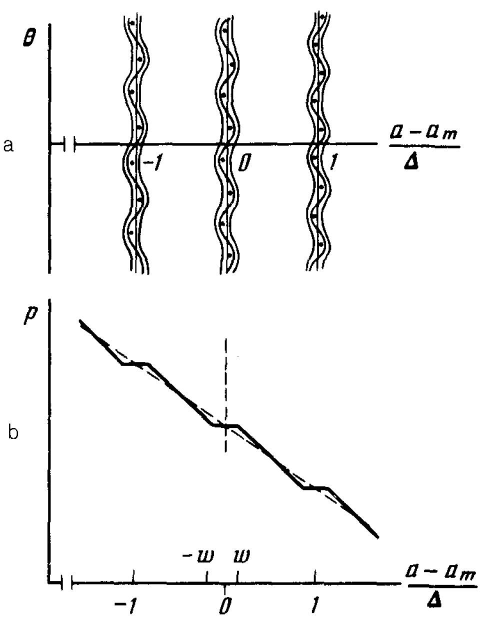

FIG. 1. a—system of magnetic islands created by a ballooning mode near resonant surfaces; b—the original and modified pressure profiles for the problem of neutral stability.

resonant surfaces, $\Delta = 1/nq'$, where $q(a)$ is the safety factor, and the typical size of a linear perturbation, $S/nq' = a/m$, where S is the shear. Below, all linear dimensions, including the width w, are divided by Δ.

Using the Connor-Hastie-Taylor method, and making use of the coordinate dependence of the parameter $\alpha = 8\pi p' R q^2/B^2$, we can write the equation

$$\frac{d}{dy}(1+S^2y^2)\frac{dF}{dy} = -\alpha(\cos y + Sy\sin y - \alpha/2)(F-1) + \alpha SJ/2 \quad (1)$$

which generalizes to the quasilinear case the equation of the linear theory of ballooning modes. Here $F(y)$ is the Fourier transform of a harmonic of the radial displacement of the plasma, and I and J are integral operators which describe the quasilinear effect. They are given by

$$I(y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt F(t)\left(\sum_{k=-\infty}^{\infty}\int_{k-w}^{k+w}\cos xt \cos xy\, dx\right),$$

$$J(y) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\cos(k+w)y\int_{-\infty}^{+\infty} F(t)[\cos(k+w-1)t - \cos(k+w+1)t]\,dy. \quad (2)$$

Self-adjoint integro-differential equation (1) determines the condition for the minimum of 2 functional whose form can be found from (1) and (2).

At small values $S \sim \alpha^2 \ll 1$, Eq. (1) can be solved by an averaging method. After the oscillatory part, $\widetilde{F}$, is eliminated, we find

$$\frac{d}{dz}(1+z^2)\frac{dF_0}{dz} + \frac{\lambda^2}{1+z^2}F_0 = \frac{\alpha^2}{2\pi S^2}\left[2\int_0^{\hat{w}} dx \cos xz \int_0^{\infty} F_0(t)\cos xt\, dt - \right.$$

$$\left. - \cos\hat{w}z\int_0^{\infty}\frac{tF_0}{1+t^2}\sin\hat{w}t\,dt - \frac{z\sin\hat{w}z}{1+z^2}\int_0^{\infty} F_0\cos\hat{w}t\,dt\right], \quad (3)$$

$$F_0 = F - \widetilde{F}, \quad \hat{w} = w/S, \quad \lambda^2 = \frac{\alpha^2}{S}\left(1 - \frac{7}{32}\frac{\alpha^2}{S}\right), \quad z = Sy.$$

Strauss[1] has analyzed Eq. (3) in detail for the case without the quasilinear terms.

In the region of the linear instability, $\lambda^2 > 1$, the quasilinear terms make it possible to find a solution of (3) which satisfies the boundary condition $F_0(\infty) = 0$. If $\lambda^2 - 1 \ll 1$, this can be done under the condition

$$\frac{2\alpha^2}{\pi S^2}\hat{w}[1 + K_0(\hat{w})] \approx \frac{2\alpha^2}{S^2\pi}\hat{w}\left(0{,}43 + \ln\frac{2}{\hat{w}}\right) = \frac{\pi}{4}(\lambda^2 - 1), \quad (4)$$

where K_0 is the modified Bessel function.

According to this model, the region of the linear instability is bounded by two conditions: $\sqrt{2}/8 < S/\alpha^2 - 1/2 < \sqrt{2}/8$. The middle of this region, which corresponds to the maximum linear growth rate, yields an estimate of the maximum width of the islands for $S/\alpha^2 = 7/16$:

$$\frac{w}{S}\left(0.43 + \ln\frac{2S}{w}\right) = S/13, \tag{5}$$

This width is small in comparison with the length scale (S) of a linear perturbation because of the assumption $S \ll 1$ and also because of the numerical factor.

The result derived above is evidence of a progressive degradation of the confinement due to the formation of magnetic islands as the plasma pressure is raised above the critical level with respect to ballooning modes. This result correlates with experimental results with the ASDEX tokamak,[5] where a smooth degradation of the parameter β was observed in limiting-pressure operation. On the other hand, the possibility of a saturation of ballooning modes at a low level, established in the present study, casts doubt on whether the pressure limits which are presently being calculated from the linear theory are in fact absolute limits. The question requires more-detailed study, both theoretical and, especially, experimental.

The quasilinear equation derived here for ballooning modes makes it possible to study the entire ranges of parameters S and α, without the limitations adopted above. In a variational formulation, it would be possible to derive analytic criteria for the saturation of the ballooning modes.

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