# Heat-transfer limitation as a cause of turbulence in plasma flows

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A limitation on heat transfer drives a hydrodynamics instability of plasma flows. Turbulent viscosity stops the instability.

A limitation on heat transfer is known to occur in a laser plasma when the pump beam is sufficiently intense. The hydrodynamic calculations which incorporate this limitation use the model of laminar plasma flows. We show in the present paper that it is simple to arrange conditions such that an instability is driven in a laser plasma by an anomalous heat transfer, and this instability produces a turbulence. The implication is that the conventional laminar-flow model is of limited applicability to the flows of a laser plasma.

We describe the limitation on heat transfer in the standard way, by means of a limitation factor $f$, writing the electron heat flux $\mathbf{q}$ as

$$\mathbf{q} = - f n_e \kappa T_e v_{Te} \vec{\nabla} T_e | \vec{\nabla} T_e |^{-1}, \tag{1}$$

where $n_e$, $T_e$, and $v_{Te}$ are the electron density, temperature, and thermal velocity; and $\kappa$ is the Boltzmann constant. The equations describing the plasma flow when ordinary viscosity is ignored are

$$\frac{\partial n_e}{\partial t} + \mathrm{div}\, n_e \mathbf{u} = 0, \quad \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\vec{\nabla})\mathbf{u} = - \frac{z}{n_e m_i} \vec{\nabla} n_e \kappa T_e ,$$
$$n_e \left( \frac{\partial T_e}{\partial t} + \mathbf{u}\vec{\nabla} T_e + \frac{2}{3} T_e \mathrm{div}\, \mathbf{u} \right) + \frac{2}{3} f \mathrm{div} \left( n_e T_e v_{Te} \vec{\nabla} T_e | \vec{\nabla} T_e |^{-1} \right) = 0, \tag{2}$$

where $\mathbf{u}$ is the flow velocity, and $z$ and $m_i$ are the ion charge and mass. The scale time $(\tau)$ of the changes in the hydrodynamic quantities must be long in comparison with the scale time for the onset of the anomalous heat flux in (1), and the corresponding scale length $l$ must be short enough that the transport is in fact anomalous, rather than classical. If the heat transfer is limited by an ion acoustic turbulence,[1] the latter requirement means

$$\tau \gg \omega_{Le}/\omega_{Li}^2 \text{ and } l < L_{\text{thresh}} \simeq v_{Te}\omega_{Le}/\omega_{Li}v_{ei},$$

where $\omega_{Le(i)}\, v_{e(i)i}$ are the electron (ion) plasma frequency and electron-ion (ion-ion) collision rate. Assuming that there exist solutions of Eqs. (2) which describe an unperturbed plasma flow with scale values $l$ and $\tau$, we examine the stability of such a flow with respect to small-scale $kl \gg 1$, rapidly varying $\omega\tau \gg 1$ perturbations. We wish to stress that we are considering current-free plasma flows, so that our approach differs from that which might be taken in the case of Joule heating with an anomalous resistance.

TABLE I.

| Heat transfer | | Instability condition |
|---|---|---|
| Weakly ionized plasma; collisions of electrons with neutrals | $\mathbf{q} = -K\dfrac{n_e T_e}{\nu_{en}}\vec{\nabla} T_e$ | $q \geqslant n_e \kappa T_e v_s$ |
| Fully ionized plasma; collisions of electrons with ions | $\mathbf{q} = -K\dfrac{n_e T_e}{\nu_{ei}}\vec{\nabla} T_e$ | $\dfrac{q}{2\Lambda} \geqslant n_e \kappa T_e v_s$ [1] |
| Laser plasma; limitation on heat transfer | $\mathbf{q} = -f n_e \kappa T_e v_{Te} \vec{\nabla} T_e \lvert \vec{\nabla} T_e \rvert^{-1}$ | $f \geqslant \dfrac{v_s}{v_{Te}}$ |
| Extreme temperatures; radiative heat transfer | $\mathbf{q} = -K T_e^{\nu} n_e^{\mu} \vec{\nabla} T_e$ | $\lvert \mu \rvert q \geqslant n_e \kappa T_e v_s$ |
| General case | $\mathbf{q} = \Phi(n_e, T_e, \vec{\nabla} T_e)\vec{\nabla} T_e$ | $\left\lvert \dfrac{\partial \ln \Phi}{\partial \ln n_e} \right\rvert q \geqslant n_e \kappa T_e v_s$ |

[1] $\Lambda$ is the Coulomb logarithm.

For perturbations $\delta n$, $\delta T$, $\delta \mathbf{u} \propto \exp(-i\omega t + i\mathbf{kr})$, we find from the linearized version of Eqs. (2) the results

$$\delta \mathbf{u} = (\mathbf{k}\omega'/k^2)(\delta n/n_e) \text{ and } \delta T = (T_e/\omega_s^2)(\omega'^2 - \omega_s^2)(\delta n/n_e),$$

where $\omega' = \omega - \mathbf{ku}$, $\omega_s = kv_s$, and $v_s = \sqrt{zT_e/m_i}$, and we find the dispersion relation

$$(\omega'^2 - \omega_s^2)(\omega' + F\omega_s \cos\theta + \tfrac{2}{3} iFkL\omega_s \sin^2\theta) = \tfrac{2}{3}\omega_s^2(\omega' - F\omega_s \cos\theta).$$

Here $\theta = \sphericalangle \mathbf{k}, \vec{\nabla} T_e$, $F = (\omega_{Le} f/\omega_{Li})L = |\vec{\nabla} \ln T_e|^{-1}$, and $kL \gg 1$. For the two roots of the dispersion relation $\omega_\pm = \mathrm{Re}\,\omega_\pm + i\gamma_\pm$ corresponding to an instability we have

$$\gamma_\pm \simeq \frac{\pm 2FkL\sin^2\theta\,(F\cos\theta \mp 1)\omega_s}{9(F\cos\theta \pm 1)^2 + 4(kLF\sin^2\theta)^2}\ . \tag{3}$$

According to (3), an instability can occur only if $F > 1$, when $q$ exceeds $n_e \kappa T_e v_s$. This instability, which results from a large heat flux, is to some extent similar to the current-driven hydrodynamic instability of a weakly ionized plasma which was discussed in Ref. 2, if we understand $q/n_e\kappa T_e$ as the electron drift velocity. The instability results from the density dependence of the heat flux ($q \propto n_e$); in the case of classical transport, the behavior $q \propto n_e$ would also hold in a weakly ionized plasma.

With $\sin^2\theta = \frac{3}{2}(F+1)(FkL)^{-1}, \ll 1$ the quantities in (3) reach a maximum value $\gamma_m(k)$ given by $\gamma_m = \alpha\omega_s$, $\alpha = (F-1)/6(F+1)$, where $\alpha$ varies form 0 ($F = 1$) to 1/6 ($F \gg 1$). We also note that expression (1) corresponds to an effective collision rate $\nu_{\rm eff} \sim v_{Te}^2/FLv_s$, and the condition for the applicability of the hydrodynamic approach, $kv_{Te} < \nu_{\rm eff}$, gives us a restriction on the wave number: $k < k_{\max} \equiv \omega_{Le}/F\omega_{Li}L$. We then find the following estimate of the maximum growth rate: $\sim \alpha v_{Te}/FL$. This maximum would be reached at $k \sim k_{\max}$.

The onset of the instability is accompanied by perturbations of the average plasma density, the temperature, and the velocity. A small-scale convection arises and leads to a mixing; the turbulent plasma flow is described by the equations

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,\rho\mathbf{V} = 0,\quad \frac{\partial}{\partial t}\rho V_p + \frac{\partial}{\partial r_j}\rho V_p V_j + \frac{\partial}{\partial r_j}(n_e\kappa T_e\delta_{pj} + \sigma_{pj} - \rho U_p U_j) = 0,$$

$$\frac{\partial}{\partial t}\left[n_e\kappa T_e\left(\frac{3}{2} + \frac{1}{2}\left|\frac{\delta n}{n_e}\right|^2\right) + \rho\frac{V^2 - U^2}{2}\right] + \frac{\partial}{\partial r_j}V_j\left[n_e\kappa T_e\left(\frac{3}{2} + \frac{1}{2}\left|\frac{\delta n}{n_e}\right|^2\right) + \rho\frac{V^2 - U^2}{2}\right]$$

$$+ \frac{\partial}{\partial r_j}V_p\left(n_e\kappa T_e\delta_{pj} + \sigma_{pj} - \rho U_p U_j\right) + \frac{\partial}{\partial r_j}[q_j + U_p(\rho U_p U_j - \sigma_{pj})] = 0, \tag{4}$$

which are found from (2) by taking an average over the small temporal and spatial scales of the fluctuations. In (4) we are using the notation $\rho = n_i m_i$, $\mathbf{V} = \mathbf{u} + \mathbf{U}$,

$$\left|\frac{\delta n}{n_e}\right|^2 = \frac{1}{n_e}\int d\mathbf{k}\,|\delta n(\mathbf{k})|^2, \quad U_j = \frac{1}{n_e}\int d\mathbf{k}\,\frac{d\omega_s}{dk_j}\left|\delta n(\mathbf{k})\right|^2,$$

$$\sigma_{pj} = \kappa T_e \int d\mathbf{k}\frac{k_i k_j}{k^2}\left|\delta n(\mathbf{k})\right|^2.$$

Equations (4) evidently apply to turbulent flows with a heat transfer described by a variety of laws, not exclusively that in (1). The corresponding possibilities for the onset of turbulence for various heat-transfer laws are illustrated in Table I. According to (4), the quantity

$$n_e \kappa T_e\left(\frac{3}{2}+\frac{1}{2}\left|\frac{\delta n}{n_e}\right|^2\right)+\rho\frac{V^2-U^2}{2}$$

has the meaning of an internal energy, and the turbulent plasma flow is characterized by a stress tensor $n_e\kappa T_e\delta_{ij}+\sigma_{ij}-\rho U_i U_j$, and a heat flux $q_j+U_p(\rho U_p U_j-\sigma_{pj})$ which can lead to a further anisotropy of the transfer, as a result of the fluctuations that arise.

Turning to the mechanisms which might stop the instability, we first note that oscillations will not grow if the growth rate of the instability is lower than the rate ($\gamma_s$) of the damping by the ordinary viscosity ($\gamma_s \simeq \nu_{ii} z T_e/T_i$ at $\nu_{ii}<\omega_s, \gamma_s \simeq k^2 v_{Ti}^2/\nu_{ii}$ at $\nu_{ii}>\omega_s$), which was ignored above but which may turn out to have a stabilizing effect. Under the condition $\alpha < T_i/zT_e$, for example, in the wavelength interval $(\alpha\nu_{ii}/v_s)(zT_e/T_i)<k<(\nu_{ii}/\alpha v_s)\times(T_i/zT_e)$, with $\gamma_m<\gamma_s$, there is no buildup of waves. Since $L^{-1}<k<k_{\max}$, we find a condition for the stability of the ground state: $(v_s/\alpha\nu_{ii})(T_i/zT_e)>L>(\omega_{Le}/F\omega_{Li})(\alpha v_s/\nu_{ii})\times(zT_e/T_i)$. This condition can be arranged if the condition $\alpha(zT_e/T_i)<\sqrt{\omega_{Li}F/\omega_{Le}}$ holds; if it does not, the ordinary viscosity will not stop the instability. The excitation of fluctuations gives rise to a turbulent viscosity, according to (4), which stops the instability. A comparison of the buildup rate with the rate of damping by the turbulent viscosity shows that a stabilization occurs at $|\delta n/n_e|^2\sim 1$, i.e., at a fluctuation level at which mixing is important. Here we would have $U\sim v_s$, $\sigma\sim n_e\kappa T_e$, and the energy of the fluctuations would be comparable to the thermal energy of the plasma, according to (4). The pressure due to the fluctuations would be comparable to the ordinary (gas-dynamics) pressure; it would become anisotropic. Finally, at $F\sim 1$, the anomalous heat flux associated with the fluctuations would be comparable to that in (1). We conclude from all these arguments that the onset of convection can affect the nature of plasma flows, causing them to differ from laminar flows.

In summary, a limitation on heat transfer is a new cause of the onset of turbulence in the gas dynamics of a laser plasma.

[1]V. Yu. Bychenkov and V. P. Silin, Zh. Eksp. Teor. Fiz. **82**, 1886 (1982) [Sov. Phys. JETP **55**, 1086 (1982)].

[2]B. Milich and A. A. Rukhadze, Zh. Tekh. Fiz. **38**, 229 (1968) [Sov. Phys. Tech. Phys. **13**, 166 (1968)].

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