Are black holes quantized?

V. F. Mukhanov
Institute of Nuclear Studies, Academy of Sciences of the USSR

(Submitted 21 April 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **44**, No. 1, 50–53 (10 July 1986)

The spectrum of a quantized black hole is derived. A definition of the entropy of quantized black holes is suggested. This definition agrees with the results available. Some possible consequences of the new results for Hawking radiation and small black holes are pointed out.

In the absence of a satisfactory quantum theory of gravitation, we can only hypothesize about possible physical consequences of such a theory. Studies in this field are important since they may cast light on the structure of a future fundamental theory. In the present letter we assume that a black hole is quantized, and we derive its spectrum on the basis of some simple heuristic considerations.

Spectrum. Let us assume that the mass, angular momentum, charge, and surface area of a black hole are quantized in the sense that they are functions of a discrete variable n:

$$M_n = M(n),\ \Omega_n = \Omega(n),\ \Phi_n = \Phi(n),\ A_n = A(n).$$

According to the correspondence principle, for large black holes ($n \gg 1$), the *principal* characteristics of Hawking radiation[1,2] must be reproduced. Hawking radiation is treated here as the result of a spontaneous transition of a black hole from level n to one of the closest levels, say, $n-1$. In other words, the frequency $\omega_{n,n-1}$, the angular momentum m, and the charge e of the field quantum emitted in the transition from level n to level to $n-1$ must satisfy the relation

$$\omega_{n,n-1} - e\Phi_n - m\Omega_n = \alpha T, \tag{1}$$

which characterizes a "typical" quantum of the thermal Hawking spectrum. Here T is the temperature of the black hole, and α is a numerical factor. (We will be using a system of units with $c = \hbar = G = k = 1$.) Using

$$\Delta M_{n,n-1} = \omega_{n,n-1}, \; \Delta\Phi_{n,n-1} = e, \; \Delta\Omega_{n,n-1} = m$$

relation (1), and the first law of thermodynamics for black holes,[3,4]

$$dM = \frac{1}{4} T dA + \Omega dJ + \Phi dQ, \tag{2}$$

we find the spectrum of the black hole in the case $n \gg 1$ to be

$$A_n = 4\alpha n. \tag{3}$$

The area of a quantized black hole is thus the Planck area multiplied by an integer (within a factor of 4α). There are no further restrictions on M, Ω, or Φ.

Entropy. It is natural to define the entropy as the logarithm of the number of possible internal configurations of a quantized black hole formed from *given* matter with *given values* of the energy, angular momentum, and charge. It is not difficult to see that the number of such configurations is equal to the number $\Gamma(M_n, \Omega_n, \Phi_n) = \Gamma(n)$, which is the number of ways in which a black hole could eventually reach level n:

$$\Gamma(n) = 2^{n-1}. \tag{4}$$

We thus find the entropy

$$S_n = \ln \Gamma(n) = (n-1)\ln 2. \tag{5}$$

The number $\Gamma(n)$ characterizes the degree of degeneracy of level n. It follows from (5) that the minimum possible change in the entropy of a black hole ($\Delta S = \ln 2$) is exactly equal to the minimum entropy (corresponding to one bit of information) which an elementary particle can carry.[6]

It follows from Hawking's results that we have[5]

$$S = \frac{1}{4} A + \text{const} \tag{6}$$

Expressions (3) and (5) agree with (6) only if we have $\alpha = \ln 2$ and const $= -\ln 2$. As a result, we have

$$A_n = 4\ln 2n, \tag{7}$$

$$S_n = \frac{1}{4}A_n - \ln 2 = (n - 1)\ln 2. \tag{8}$$

The minimum change in the surface area of a black hole is $\Delta A = 4\ln 2$, in agreement with Bekenstein's result.[3]

Radiation intensity and level width. Since the angular momentum and charge are emitted comparatively rapidly, we restrict the analysis below to the particular case of a nonrotating black hole with zero charge. For this case we find from (7)

$$M_n = \left(\frac{\ln 2}{4\pi}\right)^{1/2} \sqrt{n}. \tag{9}$$

For $n \gg 1$, the n dependence of M_n agrees with the spectrum found by Mal'tsev and Markov[7] for a quantized gravitating dust cloud.

To find the radiation rate, we need to determine the lifetime (τ_n) of a black hole in level n, which is finite because of the interaction of the black hole with the vacuum of various physical fields. We estimate this time to be

$$\tau_n \sim 1/W_n \tag{10}$$

where W_n is the imaginary part of the effective action. It is natural to suggest that W_n can be written in the form

$$W_n \simeq \beta \int_{r > r_g} (C_{ijkl}C^{ijkl} + \ldots)\sqrt{-g}\, d^4x \simeq \gamma \frac{\ln 2}{8\pi M_n} \simeq \gamma \Delta M_{n,\, n-1}, \tag{11}$$

where the restriction of the range of integration to values $r > r_g$ can be justified either by a Euclidean approach to the quantization of gravitation[8] or by causality considerations.

The width of each of the levels, W_n, is proportional to the distance between levels, $\Delta M_{n,n-1}$, and we can say that there is a well-defined structure of levels only if $\gamma \ll 1$ (if $\gamma > 1$, the levels overlap).

From (10) and (11) we find

$$\frac{dM}{dt} \simeq -\frac{2{,}8\,T}{\tau_n} \simeq -\gamma \frac{2{,}8\ln 2}{(8\pi)^2}\frac{1}{M^2}. \tag{12}$$

The M dependence of the rate at which the black hole loses mass agrees with Hawking's formula, providing further evidence in favor of our hypothesis. Comparing (12) with the corresponding expression for the intensity of the emission of quanta of a scalar field,[6] we find $\gamma_{\mathrm{sc.f}} \sim 1/30$. Since the emission brightness falls off rapidly with increasing spin of the emitted particles,[9] we do not rule out the possibility $\gamma_\Sigma = \Sigma\gamma_{\mathrm{sc.f}} + \ldots \ll 1$, and the representation of levels of a black hole is competely justified.

Consequence of the quantization of a black hole. Since, strictly speaking, spectrum

(7) was derived under the condition $n \gg 1$, we first consider large black holes. Although the emission spectrum of such black holes should agree in principal features with the thermal Hawking spectrum, it will be different from the latter in details in the case of quantized black holes (by analogy with the classical and quantum radiation of the hydrogen atom at $n \gg 1$). In the first place, the spectrum becomes a line spectrum. Second, since the radiation frequency in the case of a single-quantum decay cannot be smaller than the distance between nearest levels, there must be a pronounced (exponential?) buildup in the spectrum at wavelengths corresponding to large radii of the black hole ($\lambda > (4\pi/\ln 2) r_g$). Correspondingly, a quantized black hole will differ from a classical black hole in that it will not absorb radiation at a wavelength exceeding its own dimensions. We wish to emphasize that all these assertions hold only in the case $\gamma \ll 1$.

The conclusions regarding small black holes are more speculative. Working from the analogy with the hydrogen atom, and assuming that the spectrum (10) also holds at small values of n ($n \sim 1$), we would conclude that no black holes with a mass below $m_{\min} = (\ln 2/4\pi)^{1/2} m_{\mathrm{Pl}}$ would exist (cf. Ref. 10). The internal entropy of minimum black holes is zero [see (8)]; this is an argument in favor of their stability. Furthermore, the mass of a charged or rotating black hole would always be greater than $m_{\min}$ because of the relation $A \geqslant A_{\min} = 4 \ln 2$.

The basic ideas of this letter originated in many discussions with M. A. Markov. I also wish to thank S. M. Apenko, Ya. B. Zel'dovich, A. I. Zel'nikov, D. A. Kirzhnits, and V. P. Frolov for discussions.

[1]S. W. Hawking, Nature **248**, 30 (1974).
[2]S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[3]J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
[4]J. D. Bekenstein, Phys. Rev. D **9**, 3292 (1974).
[5]S. W. Hawking, Phys. Rev. D **13**, 191 (1976).
[6]B. S. De Witt, Phys. Rev. C **19**, 295 (1975).
[7]V. K. Mal'tsev and M. A. Markov, Preprint P-0160, Institute for Nuclear Research, Academy of Sciences of the USSR, 1980.
[8]G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2725 (1977).
[9]D. N. Page, Phys. Rev. D **13**, 198 (1976).
[10]V. P. Frolov and G. A. Vilkovisky, ICTP Preprint ICc/79/69, Trieste, 1979.

Translated by Dave Parsons