Tunneling transitions at cyclotron resonance of a free electron

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The scale time for a tunneling transition between two stable states of the motion of a free electron with a low energy at cyclotron resonance is derived.

In a remarkable recent experiment,[1] Gabrielse *et al.* have observed the first case of a bistability and a hysteresis at the cyclotron resonance of a single free electron. This effect, which is characteristic of a nonlinear oscillator, stems from the relativistic correction to the mass of the electron, which was negligible under the experimental conditions: 10^{-7}–10^{-5} (the excitation energy was $\sim$0.1–10 eV). The corresponding shift of the cyclotron frequency, however, was much greater than the linewidth ($\gamma/2\pi = 0.5$ Hz), which was determined entirely by radiative decay. The nonlinear phenomena observed at the cyclotron resonance of free electrons with low energies had been predicted earlier.[2,3]

The experimental technique of Gabrielse *et al.*[1] provides a unique opportunity for observing tunneling between two states of the cyclotron motion of an electron in a region of bistability. We studied such transitions for an anharmonic oscillator in Ref. 4, where we showed that an oscillation state with a low amplitude is metastable, and we calculated the transition probability. In the present letter we apply the results of Ref. 4 to the case of the cyclotron resonance of a free electron, and we derive the scale time (τ) for tunneling from a state with a low amplitude to one with a high amplitude.

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The Hamiltonian of an electron in a static magnetic field B and in an electric field of amplitude E which is rotating at a frequency ω can be written as follows, where we are making the first relativistic correction:

$$H = \frac{\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2}{2m} - \frac{\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^4}{8m^3c^2} + eE(x\cos\omega t + y\sin\omega t), \tag{1}$$

where e, m, and c are the magnitude of the charge of the electron, its mass, and the velocity of light, respectively; and $A_x = -By/2$, $A_y = Bx/2$, $A_z = 0$. We will not consider the motion along the magnetic field in the discussion below.

We transform to a rotating coordinate system by means of the equations

$$x = X\cos\omega t - Y\sin\omega t, \qquad y = X\sin\omega t + Y\cos\omega y \tag{2}$$

and we then make the canonical transformation

$$\begin{aligned} X &= (m\Omega)^{-1/2}(q_1 + q_2), & P_x &= (m\Omega)^{1/2}(p_1 + p_2)/2, \\ Y &= (m\Omega)^{-1/2}(p_2 - p_1), & P_y &= (m\Omega)^{1/2}(q_1 - q_2)/2. \end{aligned} \tag{3}$$

Here $\Omega = eB/mc$ is the cyclotron frequency, and P_x and P_y are the momenta which are the canonical conjugates of the coordinates X and Y. After these transformations, Hamiltonian (1) becomes $H = H_1 + H_2$, where

$$H_1 = \frac{\Omega - \omega}{2}(p_1^2 + q_1^2) - \frac{\beta}{4}(p_1^2 + q_1^2)^2 + fq_1, \tag{4}$$

$$H_2 + \frac{\omega}{2}\left(p_2^2 + q_2^2\right) + fq_2. \tag{5}$$

Here $\beta = \Omega^2/2mc^2$ and $f = eE(m\Omega)^{-1/2}$. Expression (4) is the same as that for the effective Hamiltonian for an anharmonic oscillator driven by a periodic external force.[4] The Hamiltonian H_2, which describes the motion of the center of the Larmour circle with respect to the rotating coordinate system, is unimportant to the discussion below. In contrast with the case of a one-dimensional anharmonic oscillator, in which the switch to effective Hamiltonian (4) requires discarding nonresonant oscillatory terms, the transition from (1) to (4), (5) is exact in the present case.

If the friction is small, the steady-state values of the amplitude q_1 are determined by the conditions $p_1 = 0$, $\partial H_1/\partial q_1 = 0$. In this case, the energy of the cyclotron motion is $\epsilon = \Omega q_1^2/2$. We write it as $\epsilon = n\hbar\Omega$, where n is the index of the Landau level (we are assuming $n \gg 1$). We then find an ordinary cubic equation for n, which describes the steady-state oscillations of an anharmonic oscillator:

$$\frac{n}{n_0}\left(\delta - \frac{n}{n_0}\right)^2 = \frac{4}{27}, \tag{6}$$

where

$$n_0 = \frac{3}{2^{5/3}\hbar}\left(\frac{f}{\beta}\right)^{2/3} = \frac{3}{2}\left(\frac{E}{B}\right)^{2/3}\frac{mc^2}{\hbar\Omega}, \tag{7}$$

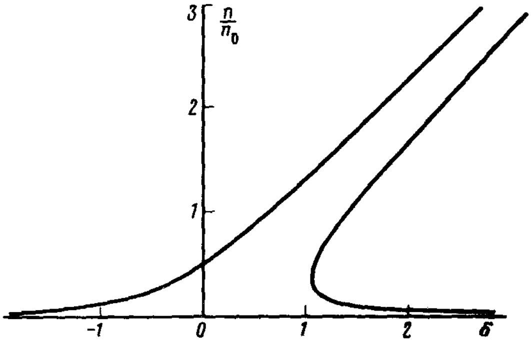

FIG. 1. Hysteresis in the dependence of the energy of the cyclotron motion on the dimensionless deviation from the resonant frequency.

$$\delta = \frac{2}{3}^{2/3} \frac{\Omega - \omega}{f^{2/3}\beta^{1/3}} = \frac{2}{3}\left(\frac{B}{E}\right)^{2/3} \frac{\Omega - \omega}{\Omega}. \tag{8}$$

There is a bistability if $\delta > 1$ (Fig. 1). The parameter n_0 is equal to the jump in n at the point at which the low-amplitude oscillations are cut off.

According to the results of Ref. 4, the scale time (τ) for a tunneling from one oscillation state to another (Fig. 1) is given in the semiclassical approximation by the expression $\tau \sim \exp[2(\Omega - \omega) J(\alpha)/\hbar\beta]$, where $J(\alpha)$ is a dimensionless function of the parameter $\alpha = f\beta^{1/2}(\Omega - \omega)^{-3/2}$, which is given explicitly in Ref. 4. The coefficient of the exponential function in the expression for τ is not known. Within this factor, we have

$$\ln\frac{\Omega\tau}{2\pi} = n_0 I(\delta), \qquad I(\delta) = 4\delta J(\alpha). \tag{9}$$

The parameters δ and α are related by $3\delta = (2/\alpha)^{2/3}$. Asymptotic expressions for the

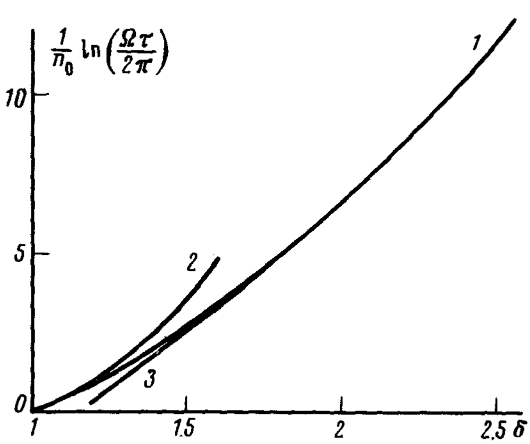

FIG. 2. Scale time for a tunneling as a function of the dimensionless deviation from the resonant frequency. 1—numerical calculation; 2—according to expression (10); 3—according to expression (11).

function $I(\delta)$ follow from the corresponding expressions derived for $J(\alpha)$ in Ref. 4:

$$I(\delta) = \frac{16\sqrt{3}}{5}\,\delta(\delta-1)^{5/4},\ \delta - 1 \ll 1, \tag{10}$$

$$I(\delta) = 6\delta\left[\ln\left(\frac{3\delta}{2^{1/3}}\right) - 1\right],\ \delta \gg 1\,. \tag{11}$$

Expressions (9)–(11) constitute the basic result of the present study. Figure 2 shows the tunneling time τ as a function of the dimensionless deviation from the resonant frequency, δ, found from expression (9) by numerical calculations. We see that asymptotic expression (11) gives a good description of the exact function $I(\delta)$ for values of δ down to those approaching unity.

For a given value of the parameter n_0, the tunneling occurs most effectively near the point of the cutoff of the low-amplitude oscillations ($\delta = 1$). The possibility of observing the transition is limited on the one hand by the uncertainty ($\Delta\epsilon$) in the measurement of the excitation energy (in the experiments of Ref. 1, this uncertainty was $\Delta\epsilon = 16$ meV, corresponding to $\Delta n = 24$) and, on the other, by the stability of the frequency of the driving alternating field.

To estimate τ we set $n_0 = 35$, $\delta = 1.2$, and $\Omega/2\pi = 1.6\times10^{11}$ Hz; we find $\tau\sim10$ s. Under these conditions the change in the excitation energy at the transition corresponds to $\Delta n\approx50$, and the necessary frequency stability is 100 Hz/s. At the value chosen for n_0, the time τ increases very sharply with increasing δ (with $\delta = 1.25$, for example, we have $\tau\sim10^5$ s). If the energy uncertainty $\Delta\epsilon$ can be reduced, the value of n_0 can also be reduced, and the frequency range in which tunneling transitions can be observed will become much broader.

We also note that the tunneling is effective when the corresponding levels of Hamiltonian (4) agree (within the radiative broadening). This circumstance may lead to oscillations of the tunneling time as a function of the parameter δ. It is not difficult to show that the period of these oscillations in δ is $(2n_0)^{-1}$.

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