Nonlinear elastoelectric interaction in non-centrosymmetrical crystals

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We consider the effect of electric fields on the propagation velocity of the elastic waves in non-centrosymmetric crystals. It is shown that the variation of the phase velocities as functions of the electric field intensity can be linear as well as nonlinear.

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The change of the velocity of propagation of elastic waves in non-centrosymmetric crystals under the influence of external electric fields is determined by the elastoelectric interaction, which leads to a linear variation of the elastic moduli (c_{ikjl}) with the electric field intensity (E_m), [1–3] namely, $\Delta c_{ikjl}=g_{mikjl}E_m$, where g_{mikjl} are the elastoelectric coefficients; the subscripts i, j, k, l, and m take on values from unity to 3.

In those cases when the applied electric field does not change the direction of the eigenvectors (the displacement vectors of the elastic waves), the change of the phase velocity of the elastic waves depends linearly on the intensity of the applied electric field. [1–5] If the electric field changes the direction of the eigenvectors, the dependence of the variation of the phase velocities on the electric field intensity is more complicated. This dependence can be determined by expressing the change of the eigenvalues of the Christoffel tensor, and consequently the phase velocities, in terms of the change of the elastic constants.

To this end, we write down the Christoffel tensor of the perturbations by the electric field in the form

$$\Gamma^{*}_{ij} = \Gamma^{0}_{ij} + \Pi_{ij},$$

where $\Gamma^{0}_{ij} = C_{ikjl} n_k n_l$ is the Christoffel tensor at $E=0$, C_{ikjl} are the elastic moduli of the crystal, and n_k and n_l are the direction cosines of the wave normal. The introduced symmetrical second-rank tensor $\Pi_{ij} = g_{mikjl} E_m n_k n_l$, where $E_m = EP_m$, and P_m are the direction cosines of the electric fields, differs from Γ^{0}_{ij} in that the eigenvalues are slightly different.

We consider the effect of the electric field on the directions of the wave normals that lie in the elastic-symmetry plane. From the solution of the Christoffel equation[6] for quasi-logitudinal and quasi-transverse elastic waves, with allowance for the elastoelectric coefficients and neglecting in the expansion the terms higher than those quadratic in the field, we obtain in general form an expression for the relative change of the elastic-wave velocity: $\Delta v/v = \alpha E + \beta E^2$, where the coefficients α and β for an arbitrary electric field lying in the elastic-symmetry plane are described, for quasi-longitudinal and quasi-transverse waves, by the expressions

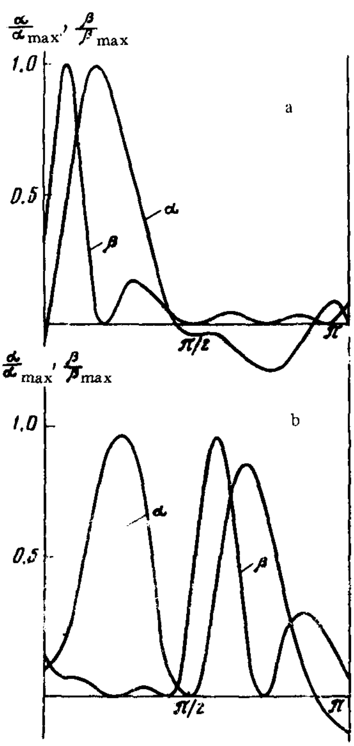

FIG. 1. Orientational dependences of the coefficients α and β in the elastic-symmetry plane for the crystal $LiNbO_3$ in a longitudinal (a) and transverse (b) electric field relative to the wave normal of the longitudinal phase velocity.

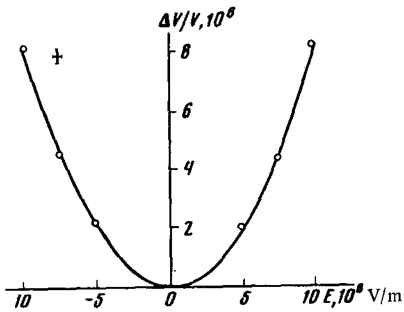

FIG. 2. Relative change of the phase velocity of a longitudinal elastic wave n ∥ [001] ($v = 7.345 \times 10^3$ m/sec) in a $LiNbO_3$ crystal as a function of the intensity of the electric field E ∥ [010].

$$\alpha = \frac{1}{2\rho v_0^2}\left[\Pi_{22} + \Pi_{33} \pm \frac{(\Pi_{33} - \Pi_{22})(\Gamma_{33} - \Gamma_{22}) + 4\Pi_{23}\Gamma_{23}}{\sqrt{(\Gamma_{33} - \Gamma_{22})^2 + 4\Gamma_{23}^2}}\right] \qquad (1)$$

$$\beta = \pm\frac{1}{\rho v_0^2}\,\frac{[\Pi_{23}(\Gamma_{33} - \Gamma_{22}) - (\Gamma_{33} - \Gamma_{22})\Gamma_{23}]^2}{\sqrt{[(\Gamma_{33} - \Gamma_{22})^2 + 4\Gamma_{23}^2]^3}}. \qquad (2)$$

Here ρ is the density and v_0 is the phase velocity of the elastic waves at $E = 0$. For crystals with symmetry $3m(3 \parallel x_3, m \parallel x_2)$, the elastoelectric interaction is described in terms of 13 independent elastoelectric coefficients. We choose, in contrast to[4,6], the independent coefficients g_{mikjl} to be such that the subscript m takes on values 3 and 2. In this case, when an electric field is applied the crystal symmetry is lowered to monoclinic in accord with the Curie principle.[7]

Figure 1 shows the computer-calculated orientational dependences of the coefficients α and β in the elastic-symmetry plane for the crystal $LiNbO_3$, using the values of g_{mikjl} measured at $f = 10^9$ Hz, in a longitudinal (a) and transverse (b) electric field relative to the wave normal to the longitudinal phase velocity. From the form of the orientational dependences we can determine propagation directions of the elastic-waves, in which the change of the velocity as a function of the magnitude and direction of the electric field is either linear or nonlinear. Thus, for $\bar{n} \parallel [001]$ and $\bar{E} \parallel [001]$, the change of the phase velocity of the longitudinal elastic waves is described by a linear term, i.e., Eq. (1) will take the form $\alpha = g_{333}/2c_{33}$ at $\beta = 0$. For the case $\bar{E} \parallel [010]$ (transverse field) and the same propagation direction as before, $\alpha = 0$ and Eq. (2) takes the form $\beta = g_{234}^2/2c_{33}(c_{33} - c_{44})$, i.e., the change of the velocity is described by a term quadratic in the field. Thus, a particular choice of the propagation direction, of the wave type, and of the direction of the applied electric field leads to different dependences of the change of the velocity of the elastic waves, measurement of which makes it possible to determine the elastoelectric coefficients.

The experimentally investigated dependence of the change of the velocity of the longitudinal wave, in the [001] direction, propagating in a $LiNbO_3$ crystal to which an electric field E[010] is applied (transverse field), is shown in Fig. 2. The measurements were carried out at room temperature on samples with dimensions 20×5×4 mm, using a scanned acoustic interferometer with linear-frequency modulation of the acoustic exciting signal and acousto-optical registration of the central frequency of the multiwave acoustic resonance. The sensitivity to relative changes of the velocity was 5×10^{-7} at a measurement time 10 sec. The hypersound was generated on a surface of a crystal placed in a coaxial resonator. The calculated value of the elastoelectric coefficient is 907±14.

The obtained nonlinear change in the velocity of the elastic wave as a function of the intensity of the applied field shows that a linear change of the elasticity of the crystals does not always lead to a linear change of the elastic-wave velocity. This fact must be taken into account not only in the determination of the elastoelectric coefficients but also in the development of various types of acoustic devices.

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