Possibility of directly observing phonon dragging by electrons in metals

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A method is proposed for a direct experimental observation of the dragging of phonons by an electric current in a metal.

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It is known that at low temperatures the mean free path in typical metals is determined by the electron-phonon collisions. The corresponding mean free path is $l_p \propto T^{-1}$ and therefore remains exceedingly small down to helium temperatures. As a rough estimate we can assume $l_p(T) \approx l_{ep}(\Theta)\Theta/T$, where $l_{ep}(\Theta) \approx 10^{-5}$—$10^{-6}$ cm is the electron mean free path relative to scattering by phonons, taken at the Debye temperature Θ. On the other hand, phonon scattering by microscopic defects of the crystal lattice (including isotopes) is proportional to the Rayleigh factor $(T/\Theta)^4$ and is therefore not very effective at low temperatures. Therefore, in samples that are not too dirty, the phonons are almost completely dragged by the electrons at low temperatures.

In this paper we discuss the following possibility of experimentally observing the phonon dragging effect. We consider two metallic plates M and M', separated by a thin dielectric layer. It is assumed that this layer is impermeable to the electrons but at the same time lets the phonon pass freely. Let a potential difference be applied to the plate M' and let a current flow. Then a dragging emf, due to the phonon pressure on the electrons, is produced in the plate M.

We consider first the case of "dirty" samples, whose resistivity in the bulky state is determined by electron scattering by crystal-lattice defects: $l_i \ll l_{ep}$, where $l_{ep}(T) \propto T^{-5}$ is the electron-phonon scattering transport length and l_i is the length of scattering by defects. It is obvious that under these conditions the electron system in the plate M is much closer to equilibrium than the phonon system, so that in the calculation of the nonequilibrium increment to the distribution function of the phonons the electrons can be regarded as being in equilibrium. (A measure of the disequilibrium of the electron or phonon system can be the velocity of the reference frame in which the total momentum of the quasiparticles is equal to zero.)

We write down the system of kinetic equations for the nonequilibrium increment to the distribution functions of the phonons $\psi(z,\mathbf{q})$ and of the electrons $\chi(z,\mathbf{p})$ in the plate M:

$$s_z \frac{\partial \psi}{\partial z} + \frac{1}{\tau_p}\psi = 0; \quad v_z \frac{\partial \chi}{\partial z} + \frac{\xi}{\tau_i}\chi = \hat{J}\psi. \tag{1}$$

The z axis is perpendicular here to the plate boundary, $\hat{J}\psi$ is the integral of the collisions of the electrons with the nonequilibrium phonons. We have left out of the collision terms those terms which are proportional to the function χ and correspond to scattering of phonons by nonequilibrium electrons and of electrons by equilibrium phonons.

The function χ satisfies the conditions of diffuse scattering of the electrons by the boundaries of the plate M. The boundary conditions for the function ψ can be assumed to be[1]: $\psi(z=0, s_z>0) = -uq_x$, $\psi(z=d,\ s_z<0)=0$, where d is the thickness of the plate M, and the drift velocity u is connected with the current density j' and the intensity of the electric field E' in the plate M': $j' = \sigma' E' = n'eu$, $\sigma' = n'e^2 p_F^{-1} l'$. Here l' is the transport mean free path of the electrons in the plate M'; the prime labels quantities pertaining to this plate.

Leaving out the calculations, we present the result for the total current flowing through the plate cross section, referred to the plate width:

$$J \approx j' \frac{n}{n'} \frac{l_i l_p}{l_{ep}} \left(1 + \frac{l_p}{d}\right)^{-1} \left[1 + \frac{l_i}{d \ln(l_i d^{-1} + e)}\right]^{-1}. \tag{2}$$

This result is valid under conditions when the resistance of the external circuit is small in comparison with the resistance of the plate M. In the opposite limiting case, the total current through the plate cross section is equal to zero and an electric polarization field is produced, with an intensity

$$E \approx E' \left(\frac{n}{n'}\right)^{1/3} \frac{l'}{l_{ep}} \left(1 + \frac{d}{l_p}\right)^{-1}. \tag{3}$$

The foregoing results have a simple physical meaning. We introduce the effective electric field E_{eff} which corresponds to the force of the phonon pressure on the equilibrium electrons at the boundary of the dielectric layer: $eE_{eff} \approx p_F u l_{ep}^{-1}$. This expression can be easily obtained by calculating the momentum transferred to the nonequilibrium phonons by the electrons. (It must be recognized here that the phonon pressure force acts only on the electrons in the region where the Fermi distribution is smeared out, whereas the field E_{eff} acts on all the electrons.

We consider first the case of a sufficiently bulky plate. Let, for example, $d \gg l_i \gg l_p$. In this case the field E_{eff} acts in a layer of thickness l_p and the electric current flows in a thicker layer of thickness l_i. Since l_p plays in this case the role of the electron mean free path, the total current can be estimated at $J \approx ne^2 p_F^{-1} l_p E_{\text{eff}} l_i$, which coincides with the factor preceding the brackets in (2). In the estimate of the polarization field E it is necessary to take into account the fact that a "countercurrent" flows through the major part of the plate cross section and cancels out the current J. From the current-cancelation condition: $ne^2 p_F^{-1} l_i E d = J$ follows expression (3) at $d \gg l_p$. In the case of a sufficiently thin plate ($d \ll l_p, l_i$), obviously, the total current is $J \approx ne^2 p_F^{-1} d E_{\text{eff}} d$, and the field is $E \approx E_{\text{eff}}$, in full agreement with formulas (2) and (3).

The foregoing analysis is based essentially on the assumption that the electrons in the plate M are much closer to the equilibrium state than the phonons passing through the dielectric interlayer. As shown by a detailed analysis, this assumption is valid not only in "dirty" samples ($l_{ep} \gg l_i$), but also in the general case, at any ratio of l_i and l_{ep}. The point is that when an electric current is produced under the influence of the phonon-pressure force, the effective electron mean free path is $l_{\text{eff}} \ll l_{ep}$. This is clear from the qualitative reasoning advanced above: l_{eff} coincides with the smaller of the lengths l_p, l_i, and d, whereas $l_p l_{ep}^{-1} \approx (T/\Theta)^4 \ll 1$. Therefore formulas (2) and (3), following the substitution $l_i^{-1} \to l_i^{-1} + l_{ep}^{-1}$, are valid in order of magnitude at any ratio of the probabilities of the electron-phonon and electron-impurity scattering. We note that the obtained formulas make it possible to determine directly the phonon mean free path in metals—from the character of the temperature dependence and, in particular, from the dependence of the emf on the plate thickness [see (3)]. This effect can depend qualitatively on the character of the electron spectrum. Thus, if electron conduction predominates in one of the plates and hole conduction in the other, then the directions of the currents (and the signs of the emf) will be reversed. In metals with open Fermi surfaces, the directions of the currents can depend on the orientation of the crystal axis relative to the surface of the plate and of the electric field.

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[1] Formulas (2) and (3) are of the correct order of magnitude of the probability of the passage of a phonon through the dielectric interlayer is not small compared with unity. On the other hand, the assumption that the phonons are diffusely scattered by the other plate boundary is not essential, for specular reflection leads only to a change of the numerical coefficients (which are of the order of unity) in these formulas.