A new diffraction effect—anomalous diffraction of glancing electromagnetic waves by a flat interface of transparent dielectrics

M. P. Kolomeev, P. N. Svirkunov, and S. S. Khmelevtsev

Institute of Experimental Meterology
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A new phenomenon has been observed in experiment, namely an anomalous increase of the amplitude of waves diffracted in an optically more dense medium at the total-internal-reflection angle. A theory of the effect is constructed and agrees satisfactorily with the experimental data.

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It is known[1] that the problem of the diffraction of glancing waves has been solved mainly only in the impedance approximation. Some aspects of the diffraction from a dielectric half-plane and the interaction of optical waves with weakly diffracting objects were investigated by Lenk.[2]

In this paper we investigate the singularities of the diffraction of glancing waves by the interface of transparent dielectrics. A new effect has been observed—an anomalous increase of the amplitude of waves diffracted in an optically denser medium at the angle of total-internal reflection.

The described effect was observed with the aid of the scheme illustrated in Fig. 1. The emission of a helium-neon laser with wavelength 0.63 μm was directed along the surface of a glass prism [face A in the (x,y) plane]. The length l of the face was 2–4 cm.

The diffracted radiation was observed at an angle ϕ to the z axis; this angle was equal within the limits of measurement accuracy ($\sim 0.5°$) to the total internal reflection angle of the glass. The dimension of the diffracted light field along the y axis is equal to the width of the illuminating laser beam, and the dimension along the ξ axis, which makes an angle $(\pi/2-\phi)$ to the x axis in (x,z) plane, is proportional to $R^{1/2}$ (the distance from the prism to the observation plane is $R \gg l$). The distribution of the brightness of the diffracted radiation along the axis was determined by photometry of the photographic film on which the picture was registered. Figure 2 shows the obtained distribution for

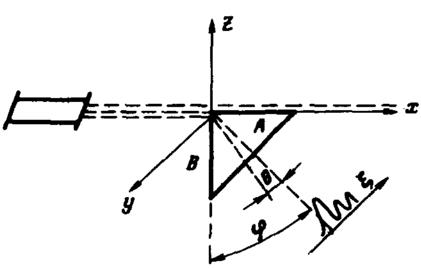

FIG. 1.

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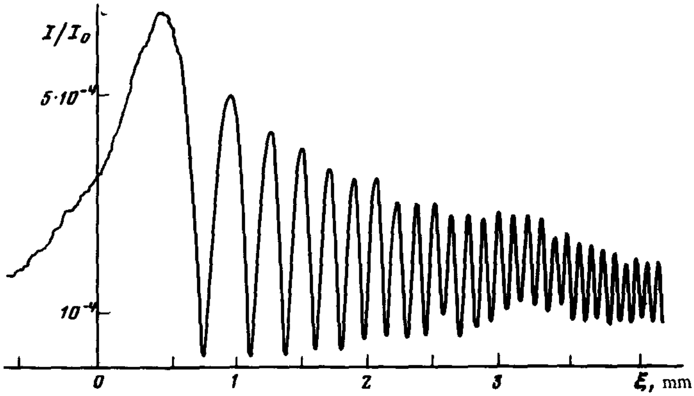

FIG. 2.

the case of a prism with a refractive index $n = 1.713$ and $R = 50$ cm. It is seen that the depth of modulation is much larger than in the case of ordinary diffraction by the edge of a screen.

The characteristics of the investigated diffracted radiation were independent of the quality of the front face B of the prism—the picture was obtained also in the case when the face B was covered with an opaque screen or when the face was made dull.

When the prism was rotated counterclockwise around the y axis, radiation reflected from the face A appeared, and the diffraction pattern in the observation plane changed smoothly into a refracted beam. When the prism was rotated in the opposite direction, the intensity of the diffracted radiation decreased by approximately one order as a result of rotation through 1°.

The effect described above was observed in different grades of glass and in quartz, and also when the prism was placed in water or in glycerine. When the refractive index of the liquid (glycerine) exceeded the refractive index of the prism material (quartz), the radiation was refracted into the optically denser liquid likewise at the angle of total internal reflection.

We proceed now to the theory of the effect, which we consider for the (x, z) plane (the planar problem). A transparent dielectric is located in the region $(x > 0,\ z < 0)$. The face $x = 0$, $z < 0$ screens the radiation. For the sake of argument we assume that the radiation is polarized in the (x, z) plane. Then Maxwell's equations reduce to a scalar wave equation for the only nonzero y component of the magnetic induction B, with B and $[1/n^2(z)]\,(\partial B/\partial z)$ continuous on the interface $[n(z) = n$ at $z < 0$ and $n(z) = 1$ at $z > 0]$. Using the Kirchhoff approximation, we assume that $x = 0$ and $z > 0$ the field corresponds to an amplitude glancing of wave $B_0\exp(ikx)$. At $x = 0$ and $z < 0$ the field is equal to zero.

Omitting the intermediate steps, we write down the solution for the region $z < 0$:

$$B = \frac{B_0 n^2}{2\pi i}\int_{-\infty}^{\infty} d\kappa\,\frac{(k+\kappa)\exp[i\kappa x - (\kappa^2 - n^2k^2)^{1/2}|z|]}{(\kappa^2 + k^2)^{1/2}[n^2(\kappa^2 - k^2)^{1/2} + (\kappa^2 - n^2k^2)^{1/2}]}\,. \quad (1)$$

The contour around the branch points k $(-k)$ or nk $(-nk)$ is drawn below (above) the real axis.

We are interested in the asymptotic limits of $kR \gg 1$ $[R = (x^2 + y^2)^{1/2}]$, which we obtain by the saddle-point method. The position of the saddle point depends on the ratio of the coordinates x and $|z|$ of the observation point. If the condition $|z|/x(n^2 - 1)^{1/2}$ is satisfied (total internal reflection angle), then the saddle point coincides with the singularity $(\kappa - k)^{-1/2}$ of the integrand. This leads to an anomalous increase of the wave amplitude. Physically it is the consequence of the stationary character of the optical path along the given direction.

Denoting by θ the angle reckoned from the total internal reflection angle, we can write the expression for the field [the principal term of the asymptotic form of expression (1)] in the form

$$B = \frac{B_o n^2}{\sqrt{2\pi}(n^2 - 1)^{1/4}} \left(\frac{2}{knR} \right)^{1/4} \int_{-\infty}^{\infty} \frac{d\nu}{\nu^{1/2}} \exp\left[-i\nu^2 - 2i\nu \left(\frac{knR}{2} \right)^{1/2} \theta \right]. \tag{2}$$

It is seen from (2) that in contrast to ordinary diffraction, where the field decreases like $R^{1/2}$, this case is characterized by a polar decrease $\sim R^{-1/4}$.

The intensity of the diffracted light is described asymptotically by expressions

$$I/I_o = \begin{cases} \left(\dfrac{2}{knR} \right)^{1/2} \dfrac{n^2}{2\pi (n^2 - 1)^{1/2}} \dfrac{1}{A} & \text{at } A >> 1 \\[2ex] \left(\dfrac{2}{knR} \right)^{1/2} \dfrac{n^2}{2\pi (n^2 - 1)^{1/2}} \dfrac{3 + 2\sqrt{2} \sin A^2}{|A|} & \text{at } A << -1 \end{cases} \tag{3}$$

$$A = \theta \left(\frac{knR}{2} \right)^{1/2}.$$

It is seen that the diffracted field has a characteristic oscillatory structure.

The results of the calculation were compared with the experimental data. The calculated values and the positions of the maxima of the diffraction pattern near $\theta = 0$ agree with the experiment within 20%.

It is seen from the foregoing that the observed effect is due to diffraction by the transparent flat interface of the two media. The source of the boundary diffracted wave is, in contrast to the ordinary diffraction, not the face but an extended section of the interface, and this is in effect the cause of the high directivity of the diffraction.

We note in conclusion that the effect described above was in all probability observed in[3], but no correct conclusion was drawn because of the inefficient experimental procedure.

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