Convective instability of a gas sphere

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The development of convective instability in a gravitating gas sphere (star) is considered. It is shown by a numerical computer solution of the two-dimensional problem that the internal layers of the medium move out to the surface, and this can lead to an increase of the luminosity of the star by many orders of magnitude.

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We consider the time evolution of convective instability of a gravitating gas sphere. The problem, with an initial two-dimensional perturbation, is solved by numerical methods with a computer.

1. Nonlinear helical MHD instability in a plasma cylinder was considered in[1–4]. It was shown that in the course of evolution of the instability the plasma is so to speak turned inside out, so that the internal layers turn out to be on the periphery. The characteristic nonlinear structures of the sections of the magnetic surfaces, obtained in[1–4], turn out to be typical also for isentropic surfaces produced when convective instability of a gas sphere (star) developes.

The equations of motion for an ideal gas are

$$\frac{\partial \rho}{\partial t} + \operatorname{div}\rho\vec{v} = 0 \quad (a) \qquad \frac{\partial N}{\partial t} + \vec{v}\,\nabla N = 0 \quad (b),$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v} = -\frac{\nabla P}{\rho} - \nabla G \quad (c), \qquad \Delta G = 4\pi\kappa\rho \quad (d), \tag{1}$$

where ρ is the density, P is the pressure, $N = P\rho^{-\gamma}$, $\vec{v}$ is the velocity, and G is the gravitational potential are solved in spherical coordinates (r, θ, ϕ) assuming axial symmetry ($\partial/\partial\phi = 0$). The initial equilibrium consideration corresponds to the Lane-Emden model[5] $P = P_0(\rho/\rho_k)^{\gamma_0}$ with $\gamma_0 = 2$ and in dimensionless variables

$$\kappa = \pi/2, \quad P = \rho^2, \quad G = -2\rho, \quad \rho = (\sin \pi r)/\pi r. \tag{2}$$

The initial velocity perturbations can be specified in the form of stationary flows $\operatorname{div}\rho\cdot v = 0$, which are described by the stream function

$$v_r = \frac{1}{\rho r^2 \sin\theta}\,\frac{\partial \Psi}{\partial \theta}, \qquad v_\theta = -\frac{1}{\rho r \sin\theta}\,\frac{\partial \Psi}{\partial r}. \tag{3}$$

The harmonics of Ψ that are regular as $r \to 0$ and satisfy the condition $v_r = 0$ at $r = 1$ can be represented, for example, in the form

$$\Psi = \lambda r^{n+1}\rho^2 \sin^2\theta\, P_n'(\cos\theta),$$

where P_n are Legendre polynomials.

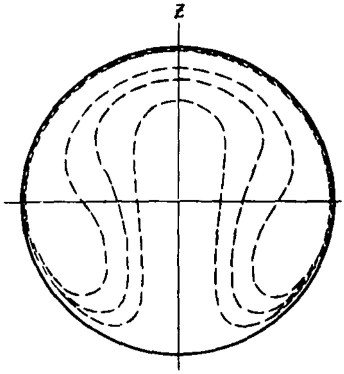

FIG. 1.

2. We consider first the kinematic model of the evolution of the first mode $n=1$ of the perturbation, assuming that the velocities do not change their spatial forms and are described by the function $\Psi=\lambda r^2\rho^2 \sin^2\theta$, where $\rho=1-r^2$.

The solution of Eq. (1b) for the function $N(r,\theta)$ that satisfies the initial condition $N=N_0(r)$ at $t=0$ is expressed by Jacobi elliptic functions[3]

$$N = N_0(u), \quad u^2 = 2/3 + \delta + (\beta - \delta)\,\mathrm{sn}^2(F - \tau),$$

$$\mathrm{sn}(F-\tau) = \frac{c\,d\,\sin\phi - sr\,(1-r^2)\,\cos\theta / \sqrt{\alpha-\delta}\,(\beta-\delta)\,\sin\phi}{1 - k^2 s^2 \sin^2\phi}, \tag{4}$$

where

$$\sin^2\phi = \frac{r^2 - \delta - 2/3}{\beta - \delta}, \qquad k^2 = \frac{\beta-\delta}{\alpha-\delta}, \qquad \tau = 2\lambda\sqrt{\alpha-\delta}\,t,$$

$$\alpha = \frac{2}{3}\cos\left(\frac{\pi}{6} + \frac{\epsilon}{3}\right), \qquad \beta = \frac{2}{3}\sin\frac{\epsilon}{3}, \qquad \delta = -2/3\cos\left(\frac{\pi}{6} - \frac{\epsilon}{3}\right),$$

$$\sin\epsilon = 1 - \frac{27}{2}\,\frac{\Psi}{\lambda},$$

c, s, and d stand respectively for $\mathrm{cn}(\tau)$, $\mathrm{sn}(\tau)$ and $\mathrm{dn}(\tau)$.

Sections through the initial spherical sections of the isentropic surfaces N

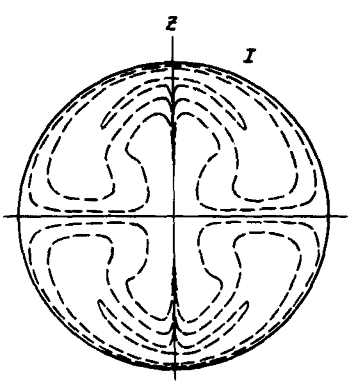

FIG. 2.

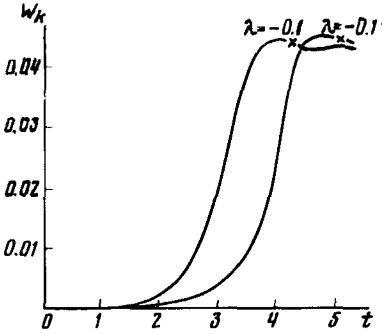

FIG. 3.

=const for the instant $\tau=1$ are shown in Fig. 1. In the course of the evolution, the internal regions of the gas sphere float to the surface, and the external regions sink to the interior.

3. The complete system of equations (1) was solved numerically with a computer for the equilibrium configuration (2) assuming the gravitational potential G to be constant. The initial velocity perturbations, satisfying the condition $\operatorname{div}\bar{v}=0$ were specified by the second-harmonic stream function ($n=2$)

$$\psi = \lambda r^3(1-r^2)\sin^2\theta\cos\theta. \tag{5}$$

In the solution of the problem we used the boundary condition $v_r(1)=0$. Figure 2 shows the sections of the isentropic surfaces for $\gamma=\frac{5}{3}$ and $\lambda=0.1$ at the instant of time $t=5.09$. Figure 3 shows plots of the kinetic energy as a function of t.

The necessary condition for convective instability is the requirement $N'(r)>0$ for the equilibrium Lane-Emden model $\gamma_0<\mu$.

In the cases considered above, the condition of convective instability was violated in the entire interval $0<r<1$, and accordingly, the instability affected the entire sphere. As a result of the development of the instability, the internal isentropic layers are carried out to the surface, and since the entropy is frozen into the medium, the internal hot layers of the gas float to the periphery. Accordingly, the surface temperature increases to $T=mN\rho^{\gamma-1}$. In the considered examples, at $\gamma=\frac{5}{3}$, the surface temperature T_1 reaches $\sim 0.2T_0$. The characteristic time of development of the instability is $\Delta t \sim R/(\gamma_0-\gamma c_0)^{1/2}$, where $c_0^2=\gamma P_0/\rho_0$. For a star such as the sun, assuming $P_0/\rho_0 \sim \kappa M_\odot/R$, we get $\Delta t_\odot \sim (\gamma_0-\gamma)^{-1/2}$ hours.

The temperature rise of the surface Σ must of necessity lead to a very strong increase of the radiation, in proportion to $T_\Sigma{}^4$.

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