Nonlinear magnetic Landau damping

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A new nonlinear effect in a compensated metal has been discovered and studied: the appearance of a magnetic Landau damping in a large-amplitude electromagnetic field.

1. Nonlinear phenomena which occur in metals as a result of the trapping of electrons by an intense electromagnetic wave and which result in a decrease in the collisionless absorption of the wave were studied in Refs. 1 and 2. In the present letter we report a study of a nonlinear effect of a completely different type: the appearance of a magnetic Landau damping with increasing amplitude of the exciting field in a geometry in which there is no such damping at small amplitudes.

Let us consider the absorption of a wave by the electrons of a metal in a strong magnetic field $\mathbf{H}_0$ which is directed along high-symmetry axis, under the conditions $\omega \ll \nu \ll \omega_c$, where ω is the wave frequency, ν is the electron collision rate, and ω_c is the cyclotron frequency. For a large-amplitude wave, the resultant magnetic field does not lie along the wave vector **k**, so that again in this geometry there is a collisionless absorption of a linearly polarized wave. To calculate it, assuming for the time being that the electron dispersion relation is quadratic and isotropic, we solve the equation of motion of an electron:

$$\dot{v}_\perp = -(eE/m - \kappa\omega_c v_z)\cos(kz - \omega t)\sin\Phi\,, \tag{1}$$

$$\dot{v}_z = -\kappa\omega_c v_\perp \cos(kz - \omega t)\cos\Phi\,, \tag{2}$$

$$\dot{\Phi} = \omega_c + (eE/mv_\perp - \kappa\omega_c v_z/v_\perp)\cos(kz - \omega t)\sin\Phi\,. \tag{3}$$

The vectors $\mathbf{H}_0$ and **k** are directed along the z axis; the wave field **E** is directed along the x axis; m is the mass of an electron; v_z and $v_\perp$ are the longitudinal and transverse components of the electron velocity; Φ is the phase of the revolution along the orbit; $\kappa = H/H_0$; and H is the magnetic field amplitude of the wave. The change in the electron energy per unit time can be put in the following form with the help of (1) and (2):

$$\dot{\epsilon} = (\omega/k)m\ddot{z}\,. \tag{4}$$

Since the Landau damping is determined by the electrons with $v_z \ll v_\perp$, Eqs. (1)–(3) simplify substantially if the wave amplitude is not too large ($\kappa \ll 1$):

$$\dot{v}_\perp = 0, \quad \dot{\Phi} = \omega_c, \quad \ddot{z} = -\kappa\omega_c v_\perp \cos(kz - \omega t)\cos\Phi\,. \tag{5}$$

Writing the coordinate as the sum of slowly and rapidly varying terms, $z = \bar{z} + a\cos\Phi$, we can solve the last equation in (5) by an averaging method.[3] Substituting the result into (4), and taking an average over the time, we find

$$\langle \epsilon \rangle = (\pi\omega m a^2/4k^2)\cdot d\delta(\Omega)/d\Omega\,, \tag{6}$$

where $\delta(\Omega)$ is the Dirac δ-function. The energy density absorbed per unit time by the electron Fermi gas is

$$Q = (\kappa^2/4)\beta k E^2\,, \qquad \beta = (3\pi/16)(ne^2/m\omega_c^2)v_F\,. \tag{7}$$

This result differs from the expression for the absorbed energy density in an oblique magnetic field in the linear case,[4] $Q_L = \tan^2\theta\cdot\beta k E^2$ in that $\tan^2\theta$ has been replaced by $\kappa^2/4$, where θ is the angle between the vectors $\mathbf{H}_0$ and **k**. A calculation shows that the relation between Q and Q_L also holds for any axisymmetric Fermi surface with the vector **k** directed along the symmetry axis. The quantity $\kappa/2$ thus serves as an effective inclination angle θ_{eff} (under our conditions, $\tan\theta_{\text{eff}} \approx \theta_{\text{eff}}$). The absorbed energy Q is proportional to the fourth power of the amplitude of the wave field. This collisionless-absorption mechanism could thus quite naturally be called "nonlinear magnetic Landau damping." It is determined by the same electrons, with $v_z = \omega/k$, as in the linear situation, but it corresponds to the simultaneous absorption of two photons.

2. This absorption mechanism does not operate for a circularly polarized wave. The term in the Hamiltonian which is nonlinear in the wave field is proportional to the square of the vector potential **A**. For a transverse wave, it can be written in the form

A_+A_-, where $A_\pm = A_x \pm iA_y$. In the semiclassical approximation, the matrix elements of this interaction are zero for a wave with a certain circular polarization. It can be shown that all the higher orders of the perturbation theory also fail to contribute to the absorption.

If, in addition to the circularly polarized wave $\mathbf{A}_2$, there is a linearly polarized wave $\mathbf{A}_1$, the Hamiltonian acquires a term proportional to $2\mathbf{A}_1\mathbf{A}_2$, which can be written in the form $A_{1x}(A_{2+} + A_{1-})$. Its matrix elements obviously do not vanish, and there may be a simultaneous absorption of one photon from the first wave and one from the second by the electron. Calculations lead to the following expression for the absorbed energy:

$$Q = \beta\{(\kappa_1^2/4)k_1E_1^2 + [\kappa_2^2k_1^2/(k_1+k_2)]E_1^2 + [\kappa_1^2k_2^2/(k_1+k_2)]E_2^2\}\ . \quad (8)$$

The first term here describes the two-photon absorption of the first (linearly polarized) wave, while the second (third) term may be interpreted as the energy of the first (second) wave which is absorbed by electrons in a magnetic field which deviates from the $\mathbf{k}$ direction because of the field of the second (first) wave. This case corresponds in a qualitative way to the application of a linearly polarized field to a metal in which a linearly polarized skin component and a circularly polarized doppleron are excited.

3. The appearance of magnetic Landau damping increases the damping of the doppleron, but in a compensated metal it has its strongest effect on the skin component. Consequences of magnetic Landau damping in an oblique field in the linear case were studied in Ref. 5. It was shown that a deviation of the field $\mathbf{H}_0$ from the normal to the surface results in a decrease in the size of the skin layer and a shift of the maximum of the smooth part of the surface resistivity $R(H_0)$ of a plate toward a stronger field. Clearly, a nonlinear magnetic Landau damping should lead to the same effect.

Figure 1 shows recordings of $R(H_0)$ of a cadmium plate for the case of a linear polarization of the exciting field with $\mathbf{k}\|\mathbf{H}_0\|C_6$ in linear and nonlinear situations. We see that in the nonlinear case R varies more slowly as a function of the field H_0, and the amplitude of the oscillations decrease significantly near the lower Doppler thresh-

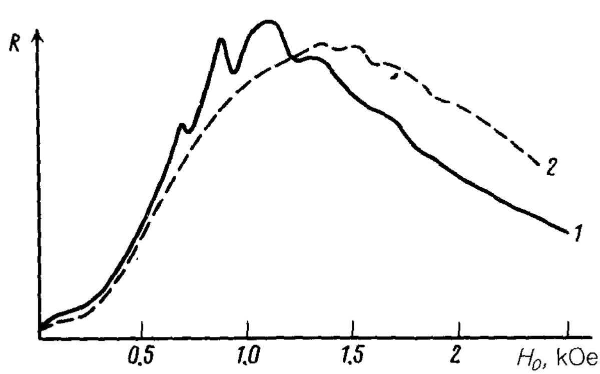

FIG. 1. Surface resistivity of a cadmium plate with a thickness $d = 1.7$ mm for field amplitudes $H = 1.5$ Oe (curve 1) and 225 Oe (curve 2). The frequency is $\omega/2\pi = 1$ kHz; $T = 1.6$ K; $\mathbf{H}_0\|\mathbf{k}\|[0001]$.

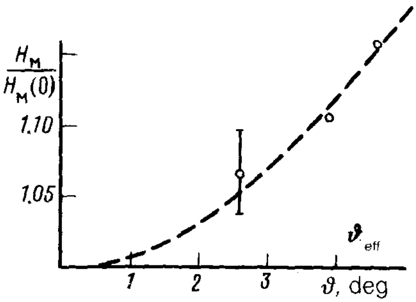

FIG. 2. Relative change in the field H_M for a cadmium plate in the nonlinear case with $\mathbf{H}_0 \| \mathbf{k} \| [0001]$ (points). The dashed line shows the change in H_M due to the tilting of the field in the linear case. The frequency is $\omega/2 = 1$ kHz; $d = 1.7$ mm; $T = 1.6$ K. For a small amplitude H and for $\mathbf{H}_0 \| \mathbf{k} \| [0001]$, we have $H_M = 1.08$ kOe.

old (H_L). The relative increase in the field H_M, which corresponds to the maximum of $R(H_0)$, with increasing amplitude of the exciting field is shown in Fig. 2. The value found for θ_{eff} from (8) is plotted along the abscissa here. In the case at hand, in which the field H_M exceeds the upper doppleron threshold H_l, the second and third terms should be omitted from (8); as a result, we find $\theta_{\text{eff}} = \kappa_1/2$. The dashed line in this figure shows the change in H_M produced as a result of deflection of a static field through an angle θ in the linear case. It follows from these that the expression $\theta_{\text{eff}} \approx \kappa_1/2$ is valid. For a circular polarization of the exciting field, the position H_M does not depend on the amplitude, in agreement with the theory.

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