The feasibility of studying electronic topological transitions by means of tunneling

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The electronic topological transitions in metals can be studied on the basis of tunneling by analyzing the specific features of the current-voltage characteristic that occur near these transitions.

It is known that a change in the topology of the Fermi surface is accompanied by the appearance of various anomalies in the thermodynamic and kinetic characteristics of the metal which undergoes such a transition.[1] The distinctive features, for example, in the thermal emf, in the conductivity, in the absorption coefficient of sound, in the superconducting-transition temperature, etc. have been studied in detail.[2–5]

In the present letter we study the special features that appear in the differential resistance of a tunnel junction whose electrodes undergo an electronic topological transition as a result of an external perturbation. For definiteness, we will analyze an electronic topological transition of the "bridge-rupture" type on the basis of a model which we have used previously.[6] In this model the electronic Fermi surface is assumed to have the shape of a hyperboloid of rotation, so that the electron-dispersion law is given by

$$\epsilon(\mathbf{p}) - \mu = \frac{\mathbf{p}_\perp^2}{2m_\perp} - \frac{p_x^2}{2m_x} - z\,. \qquad (1)$$

Depending on the sign of the parameter z which characterizes the proximity of the system to the transition, the hyperboloid is either a hyperboloid of two sheets ($z<0$) or a hyperboloid of one sheet ($z>0$), which corresponds to a closed Fermi surface or an open Fermi surface. The Green's function for such a spectrum which takes the electron scattering into account and which was found previously[6] can be written in the form

$$G^R(\mathbf{p},\omega) = \left(\omega + z - \frac{\mathbf{p}_\perp^2}{2m_\perp} + \frac{\mathbf{p}_x^2}{2m_x} - i\,\mathrm{Im}\,\Sigma^R(\omega)\right)^{-1}\,. \qquad (2)$$

In the diagram technique, the tunneling current that flows through the junction is given by a familiar expression[7]

$$I(V) = \frac{2\pi^2}{em_\perp^2 p_{x0}^2 R_n}\int d\omega\left[\tanh\frac{\omega+eV}{2T} - \tanh\frac{\omega}{2T}\right]\int\frac{d^3p}{(2\pi)^3}\,\mathrm{Im}G^R(\mathbf{p},\omega+eV)$$
$$\times\int\frac{d^3k}{(2\pi)^3}\,\mathrm{Im}G^R(\mathbf{p},\omega), \qquad (3)$$

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where R_n is the resistance of the junction.

The momentum-integrated imaginary part of the Green's function is related to the self-energy part by the self-consistency equation.[6] Taking this circumstance into account, we can write the expression for the current in the form

$$I(V) = \frac{2\tau^2}{eR_n} \int_{-\infty}^{\infty} d\omega \left[\tanh \frac{\omega + eV}{2T} - \tanh \frac{\omega}{2T}\right] \mathrm{Im}\, \Sigma^R (\omega + eV)\, \mathrm{Im}\, \Sigma^R (\omega). \tag{4}$$

We can write $\mathrm{Im}\Sigma^R (\omega,z)$ as

$$\mathrm{Im}\, \Sigma^R (\omega, z) = -\frac{1}{2\tau}[1 - \Delta(\omega + z)/\sqrt{2\epsilon_0}\,], \tag{5}$$

where

$$\Delta(x) = [[(1/2\tau)^2 + x^2]^{1/2} - x]^{1/2}, \tag{6}$$

τ is the relaxation time which is governed by the scattering of electrons by impurities, and $\epsilon_0 = p_{x0}^2/2m_x$ is the characteristic ordering energy[6] ϵ_F.

Differentiating expression (4) with respect to the voltage V for the differential resistance of the tunnel junction, which is governed by its proximity to the electron topological transition, we find

$$\frac{\delta R_{an}}{R_n} = \frac{R(V)}{R_n} - 1 = \frac{1}{4T\sqrt{2\epsilon_0}} \int_{-\infty}^{\infty} \frac{d\omega}{\cosh^2 \frac{\omega}{2T}} [\Delta(\omega - eV + z) + \Delta(\omega + eV + z)]. \tag{7}$$

If the impurity concentration is low ($T\tau \gg 1$), in which case the τ dependence of Δ can be ignored (the resulting restrictions imposed on z will be discussed below), we can write expression (7), after integrating it over ω, in the form

$$\frac{\delta R_{an}}{R_n} = \sqrt{\frac{T}{2\epsilon_0}} \left(F\left(\frac{z - eV}{2T}\right) + F\left(\frac{z + eV}{2T}\right)\right), \tag{8}$$

where

$$F(\xi) = \int_{\xi}^{\infty} \frac{\sqrt{x - \xi}\,dx}{\cosh^2 x} = \begin{cases} \sqrt{(\pi/2)}\exp(-2\xi) & \xi \gg 1 \\ \sqrt{\pi}[\zeta(1/2)(1 - 2^{-1/2}) + \zeta(-1/2)(4 - 2^{1/2})\xi] = 0.76 - 0.96\xi & |\xi| \ll 1 \\ 2|\xi|^{1/2} & \xi \ll -1 \end{cases} \tag{9}$$

Analysis of expression (8) shows that for $z \gtrsim 0$ the $R(V)$ curve has only a vaguely defined minimum at $V = 0$. At $z < 0$ expression (8) is far less trivial (Fig. 1). Here the minimum which exists at $V = 0$ gradually turns into a maximum (for $z < z^* \simeq -1.1$

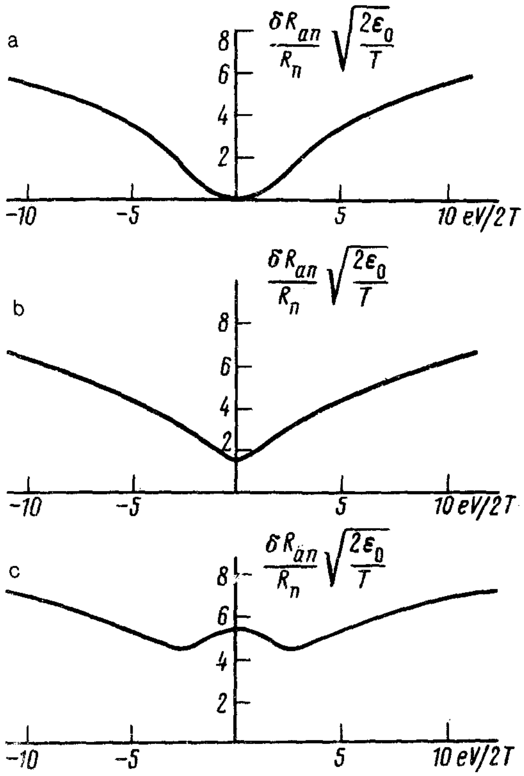

FIG. 1. Relative correction to the differential resistance of the tunnel junction versus the voltage for a constant value of the parameter z: $a - z = 2$; $b - z = 0$; $c - z = -2$.

kT) and at particular values of $\pm V_{\min}$ the $R(V)$ dependence exhibits characteristic lateral minima. At $z \ll -T$, the position of these minima can be determined analytically from the extreme value of the function $R(V)$ in (8), with use of the asymptotic representation for $F(\xi)$ [see Eq. (9)]. Assuming $z \pm eV_{\min} \gg T$ and $z \mp eV_{\min} \ll T$, we find

$$eV_{min} = \pm |z| \left\{ 1 + \frac{kT}{2|z|} \ln \frac{2\pi |z|}{kT} \right\}. \tag{10}$$

More detailed analysis of the function $\delta R_{an}/R_n$ for different values of z and V was carried out by numerical methods. The results of these numerical analyses are shown graphically in Fig. 2 for the positive and negative values of V, although the symmetry of this graph is clearly evident from (8).

At low temperatures or at high impurity concentration, we are dealing with the dirty case ($T\tau \ll 1$), so that τ^{-1} can no longer be ignored in the expression for $\Delta(x)$, but $\cosh^{-2}\omega/2T$ now varies much more rapidly than the second factor in (7) and the corresponding integral can easily be used

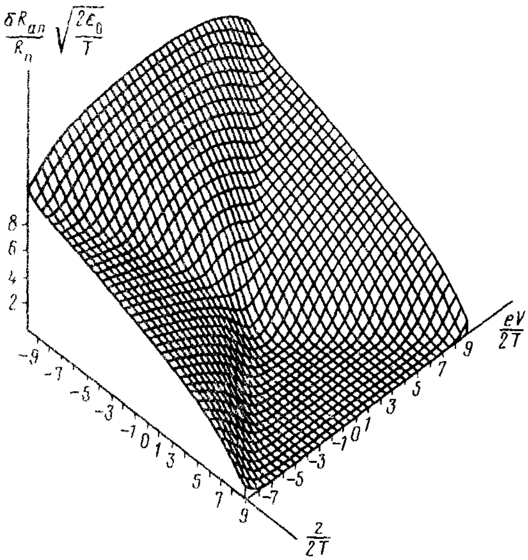

FIG. 2. Relative correction to the differential resistance of the tunnel junction versus the voltage and the parameter z which characterizes the proximity of the system to the electronic topological transition point.

$$\frac{\delta R_{an}}{R_n} = [\Delta(z+eV)+\Delta(z-eV)]/\sqrt{2\epsilon_0}\,. \tag{11}$$

This relation, which is similar to the corresponding relation for the pure junction (Fig. 2), is its limiting case upon lowering the temperature. For $z \ll -\tau^{-1}$ the position of the lateral minima in this case can also be determined analytically:

$$eV_{min} = \pm|z|\left\{1+\frac{1}{2(|z|\tau)^{2/3}}\right\}.$$

The curve for the differential resistance of a tunnel junction versus the voltage and proximity to the electronic topological transition thus has a highly peculiar feature near the electronic topological transition. At large negative values of z, the $R(V)$ curve has a minimum which corresponds to a dielectric breakdown: The closed Fermi surface in this case is near the edge of the Brillouin zone and an application of a large enough voltage ($eV\sim|z|$) "opens" it in a sense. The characteristic feature on the $R(V)$ curve is evidence of this occurrence. As $z\to z^*$, this feature vanishes, since in this region a thermal breakdown automatically occurs in this system at nonzero temperatures, even without the application of an electric field.[6] At $z>0$, the $R(V)$ curve has no anomalies, since the applied electric field no longer changes the topology of the Fermi surface.

In the case of a nonsymmetric junction, when only one of the electrodes undergoes an electronic topological transition, the current-voltage characteristic loses its symmetry relative to the change of the voltage sign. At $eV\sim \pm z$ (the sign is determined by the polarity of the voltage applied to the junction) the $\delta R(V)$ dependence in this case should have a step, instead of lateral minima.

These results obtained in the model of a topological transition of the bridge-rupture type evidently are general in nature and the $R(V)$ dependence is expected to behave in a similar manner in the "nucleation-of-voids" transitions.

At $z \gtrsim T$ the exponential asymptotic expression [Eq. (8) and also analogous expressions in Ref. 6] obtained for metal with a low impurity concentration correspond to the limit $T\tau \rightarrow \infty$.

At large enough values of z [$z \gtrsim T \ln(T\tau)$], these expressions give way to a power-law decrease of the corresponding correction.

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