Stability and kinetic effects of a standing Langmuir wave

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A standing Langmuir wave (or an infinite chain of synchronous standing solitons in the one-dimensional case) is stable and can be continuously fed by an electron beam. The interaction produces humps on the beam velocity distribution function.

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It is known[1–3] that in the one-dimensional case Langmuir waves break up into clusters (condensates). In the course of time these clusters go over into a quasistationary state and form solitons. The interaction of solitons with plasma particles was considered in[3–5]. It was shown that the motion of Langmuir solitons is made difficult by the strong deceleration by the plasma particles. In real situations the solitons can therefore be regarded as standing. It is also known[3]a that in the one-dimensional case an isolated standing soliton is always attenuated when it interacts with a beam, and the beam distribution becomes monotonically smeared out in accordance with the quasilinear theory. It might seem therefore that in plasma-beam experiments with weak instability the solitons cannot exist. Lavrovskiĭ *et al.*,[6] however, who investigated the interaction of an electron beam with a plasma in a strong magnetic field ($\omega_{pe} < \omega_{He}$, $\omega_{pe} \approx 10^9$ sec^{-1}), have observed a high level of Langmuir noise, and the spectrum of this noise, plotted over times on the order of several nanoseconds, had a line structure. They also measured the beam distribution function in short time intervals ($\sim 10^{-7}$ sec). The measurements have shown that the interaction with the plasma causes the beam distribution function to spread out greatly and to acquire several strongly pronounced humps, thus contradicting the conclusions of the quasilinear theory.

We shall show that the effects noted in[6] can be explained by assuming that the interaction of the plasma with the beam produces a standing Langmuir wave or a chain of solitons that are correlated in phase. In the presence of correlation, the beam energy is transferred to the one-dimensional solitons even if they are standing. The presence of correlation produces also humps on the distribution functions. These effects can also be attributed to a buildup of a traveling monochromatic wave,[7,8] but such a wave is unstable to self-modulation.[1,2]

If the velocity of a packet of Langmuir waves is much less than the velocity c_s of the ion sound, then the packet is described by the equation[2–5]

$$2i\frac{\partial E}{\partial t}+3\omega_{pe}r_{De}^{2}\frac{\partial^{2}E}{\partial x^{2}}+\frac{\omega_{pe}}{16\pi n_{0}Mc_{s}^{2}}|E|^{2}E=0. \tag{1}$$

Here $E\exp(-i\omega_{pe}t)$ is the electric field in the plasma, ω_{pe} is the Langmuir frequency, and $r_{De}=v_{Te}/\omega_{pe}$. This equation is fully integrable and has a nondenumerable set of integrals of motion.[9] The stationary solutions of (1) are therefore stable to small perturbation, just as the solution of the Korteweg—de Vries in the form of periodic wave is stable.[10] The only stationary solution that does not experience deceleration by the plasma particles is a solution in the form of a standing wave

$$E(x,t)=\frac{\sqrt{6}k_{0}T_{e}}{e}F(x)e^{-i\Omega t},\quad k_{0}=\frac{1}{\sqrt{6}}\frac{e}{T_{e}}E(0). \tag{2}$$

Here $\Omega=-\frac{3}{2}k_0^2r_{De}^2\omega_{pe}$, $F(x)$ is a periodic function satisfying the equation

$$k_{0}^{-2}\frac{\partial^{2}F}{\partial x^{2}}=F-F^{3},\ F=\sum_{n=-\infty}^{\infty}B_{n}e^{i\frac{2\pi n}{L}x}, \tag{3}$$

where L is the wavelength (period). In the limit of an infinite wavelength L, the Fourier series (3) goes over into a Fourier integral, $F\to\cosh^{-1}(K_0x/2)$, and (2) turns into a standing soliton with a characteristic width k_0^{-1}. Replacing F in (2) by its expansion (3), we represent the standing wave in the form of a set of traveling waves with phase velocities $L(\omega_{pe}+\Omega)/2\pi n$, where $n=0,\ \pm 1, \pm 2,\ \ldots$. If the plasma contains a beam with characteristic velocity v_B, then it suffices to confine oneself in the equation for the beam distribution function f to the contribution of only the nth wave of this set, with a phase velocity close to $v_B: L(\omega_{pe}+\Omega)/2\pi n\approx v_B$. We then have

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}+\frac{e}{m}B_{n}\sin(k_{n}x-\omega t)\frac{\partial f}{\partial v}=0. \tag{4}$$

Here $\omega=\omega_{pe}+\Omega$ and $k_n=2\pi n/L$. This equation can be easily integrated over the trajectories.[7,8] As a result we find that strongly pronounced humps appear on the initially smooth distribution function f in the course of time, as is indeed observed in experiment.

According to[7,8], if the distribution function at the resonance point has a positive derivative with respect to velocity, then the beam energy is transferred to the resonant harmonic. Since all harmonics in a nonlinear standing wave and in a soliton are interrelated,[3] an increase in the energy of one harmonic leads to an increase of the energies of all others. In contrast to an isolated solition, a chain of correlated solitons will therefore have an increment in the presence of a beam.

It is known[7,8] that if the growth rate of the beam instability is equal to γ_k, then the amplitude of the monochromatic wave increases to a value on the order of $B_n\sim(m/e)\gamma_{k_n}^2k_n^{-1}$. Then the amplitude stops growing because of the distortion of f in the region of resonant velocities. If the spread Δv_B of the beam velocities is smaller than the amplitude of the particle-velocity oscillations in the wave field, i.e., if the inequality $\Delta v_B<\gamma^2/k\omega$ is satisfied, then the resonant distortion of the function f is of the order of f itself and this case is easiest to observe in experiment. If Δv_B is not the small, then the distortion is $\Delta f \sim (\gamma/k)(\partial f/\partial v) < f$. A situation is then possible in which the beam interacts with several standing-wave harmonics with different velocities.

The formation of humps on the beam distribution function on passing through Langmuir turbulence was observed in[11], where the distortion was attributed to interaction of the beam with a set of entirely independent solitons or collapses. The authors have arrived at this incorrect explanation because they approximated the shape of the soliton by a rectangle, with an ensuing inadmissible alteration of the soliton Fourier spectrum obtained in[3].

Nonmonotonic distortion of the beam distribution function in the resonance region may also be caused by formation of a set of synchronous three-dimensional Langmuir[12] or cyclotron[13] solitons.

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