# Explosive instability of a rotating gravitating disk

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The condition under which oscillations of small but finite amplitude of a thin gravitating rotation gas disk can lead to the so-called explosive instability is derived. The disk may in this case have no Jeans instability at all.

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We have previously[1] derived a nonlinear equation that describes oscillations at a small but finite amplitude, of the velocity of a rotating gravitating disk (we use the notation of[1,2]):

$$\hat{L}^2 u_1(t,\phi) = \gamma_{k_0}^2 v_1(t,\phi) + \frac{(2-\kappa)k_0^2 \Omega_0}{\kappa_0}\left[\frac{8(2-\kappa)\Omega_0\kappa_0}{\omega_{2k_0}^2} - (3-\kappa)\right] \tag{1}$$

$$\times |v_1(t,\phi)|^2 v_1(t,\phi),$$

where

$$\gamma_{k_0}^2 = 2\pi G\sigma^0|k_0| - 2\kappa_0\Omega_0 - k_0^2 c_s^2, \quad \hat{L} \equiv \frac{\partial}{\partial t} + \Omega_0\frac{\partial}{\partial\phi},$$

$$\omega_{2k_0}^2 = m^2\Omega_0^2 \pm 2m\Omega_0\sqrt{2\Omega_0\kappa_0} + 2\Omega_0\kappa_0. \tag{2}$$

In the linear approximation, Eq. (1) describes the Toomre instability[3] of a rotating gravitating disk,[1] and the perturbed quantities increase exponentially in time.

We shall show that under the condition

$$\kappa < \frac{16\,\Omega_0\kappa_0 - 3\omega_{2k_0}^2}{8\,\Omega_0\kappa_0 - \omega_{2k_0}^2}, \tag{3}$$

the quantity $v_1(t,\phi)$ tends to infinity after a finite time.

Let us ascertain first the extent to which the condition (3) is realistic. If it is assumed that the values of the adiabat $\gamma$ are contained in the closed interval $1 \le \gamma \le 2$, then it is easily seen that for the modes $m = 1$ and $2$ the condition (3) may be satisfied. In fact, for the mode $m = 0$ the condition (3) corresponds to the inequality $\gamma < 3/2$; for the mode $m = 1$, in the case of a rigid-body rotation, $\Omega_0 = \text{const}$, the condition (3) corresponds to the requirement $\gamma < 5/4$. It is known at present (see, e.g.,[4]) that the gas in the spiral arms is in three phases. More than 70% of the matter is concentrated in the cold clouds, in

which the temperature is of the order of 100 K. About 20% is outside the clouds and has a temperature $(5-10) \times 10^3$ K. Finally, about 10% of the matter is at a temperature 10–20 K in the molecular phase. Many workers have shown (see[4] and the literature cited therein) that the perturbations propagate in these three phases isothermally, corresponding to[1] $\gamma \approx 1$. The condition (3) is therefore perfectly realistic for the modes $m = 0$ and 1.

In the particular case $m = 0$, Eq. (1) takes the form

$$\frac{\partial^2 v_1}{\partial t^2} = [\gamma_{k_0}^2 + 3 \frac{\Omega_\circ}{\kappa_\circ} (2 - \kappa)(\frac{5}{3} - \kappa) k_\circ^2 |v_1|^2] v_1 \tag{4}$$

We multiply (4) term by term by $\partial v_1/\partial t$ and integrate twice with respect to $t$. Omitting hereafter the subscript "1" of the symbol $v$, we get

$$t - t_\circ = \int_{v_\circ}^{v} [w_\circ(r) + \gamma_{k_0}^2 v^2 + A v^4]^{-1/2} dv, \tag{5}$$

where

$$A = \frac{3}{2} \frac{\Omega_\circ}{\kappa_\circ} (2 - \kappa)(\frac{5}{3} - \kappa) k_\circ^2 > 0$$

at $\kappa < 5/3$; $v_0$ is the initial perturbation of the velocity at the instant of time $t_0$; $w_0(r)$ is an arbitrary function of $r$.

In the general case, the integral in (5) is expressed in terms of an elliptic integral. To explain the character of the solution, we consider some particular case in which the integral (5) can be easily evaluated. Set, for example, $w_0(r) \lesssim \gamma_{k_0}^2 v_0^2 \ll A v_0^4$, then

$$v/v_\circ = [1 - \sqrt{A}\, v_\circ (t - t_\circ)]^{-1}. \tag{6}$$

It is seen from (6) that after a finite time $(t - t_0) \to (\sqrt{A} v_0)^{-1}$ the velocity perturbation tends to infinity. It is this persistent growth of the perturbation which characterizes the so-called "explosive" instability, which can initiate star production in a gas disk. We note that this instability is not of the Jeans type and can develop in regions much smaller than the Jeans dimension. The disk may in this case not be subjected to the Jeans instability et al., i.e., $\gamma_{k_0}^2 < 0$; it is necessary then, however, to satisfy the condition $|\gamma_{k_0}^2| \ll \Omega_0^2$.[1]

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[1] At $T > 10^4$ K, intense radiation of hydrogen takes place in the Ly – $\alpha$ line. Excitation of the lower level of carbon is the principal cooling process at the temperature $T \approx 100$ K. [4]

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