Development of convective instability in a rotating gaseous disk

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A mechanism by which a spiral structure is formed in a gravitating rotating gaseous disk is analyzed. Caused by a nonlinear development of the convective instability, this mechanism may be responsible for the origin of spiral galaxies.

The development of gasdynamic instabilities apparently is the most simple example that can be used to illustrate the ideas advanced by Prigozhin[1] on the role of self-organization in various physical phenomena. Even if the entropy is conserved, the development of an instability is an irreversible process in which the system undergoes a transition from the equilibrium, though unstable, initial state to the final state driven by small, random perturbations. The inverse transition physically cannot occur. The dissipative processes in this case account for the relaxation of the final state to the new dynamic equilibrium.

1. A nonlinear development of convective gasdynamic instabilities has several common properties,[2] allowing a variety of self-organization processes in the gas and plasma configurations to be interpreted from a single point of view. One of the characteristic features of the development of an instability is the formation of narrow "inversion" layers with concentrated parameters, which are formed along the separatrix surfaces of the vortex tubes of the eigenfunctions of the linear problem for the velocity field. In the presence of rotation, the inversion layers which are formed acquire a spiral structure with "sleeves" whose number is equal to the azimuthal model m of the eigenfunction. The spiral pattern which forms in this manner (in particular, for $m = 2$) is similar in many ways to the spiral galaxy with a bridge.[3]

In the present letter we consider the formation of a spiral structure in a gravitating rotating gaseous disk as a result of the development of a convective instability. To describe the dynamics of the process, we use a system of dissipative-free gasdynamic equations

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{v} = 0, \quad \rho \frac{d\mathbf{v}}{dt} = - \nabla p - \rho \nabla \phi, \quad \frac{d}{dt} p \rho^{-\gamma} = 0. \tag{1}$$

In the initial steady state, the gravitational attraction is assumed to be balanced by the centrifugal force and the thermal repulsion force

$$v_\varphi^2 / r = p'(r) / \rho(r) + \phi'(r). \tag{2}$$

2. In the linear stage, the development of an instability is described by a system of linearized equations (1) which for perturbations $\sim \exp i(m\varphi + \omega t)$ reduce to a single equation for the function $f(r) \sim r v_r / y$ if the gravitational-field perturbation is ignored.

$$\left(\frac{\gamma p y^2}{rs} f'\right)' - \left\{\frac{\rho y^2}{r} - \frac{2\rho v_\varphi}{r^3}\left[(rv_\varphi)' - \frac{2m^2 v_\varphi \gamma p}{\rho s r^2}\right] - \frac{p'}{\rho r}\left(\rho' - \frac{m^2 p'}{s r^2}\right)\right.$$
$$\left. - \left[\frac{y^2}{rs}\left(p' - \frac{2m v_\varphi \gamma p}{y r^2}\right)\right]'\right\} f = 0. \tag{3}$$

Here $y = \omega + m v_\varphi / r$, $s = \gamma p m^2/\rho r^2 - y^2$, and γ is the adiabatic index.

In the limit $y \to 0$, according to (3), we have

$$\left(\frac{\rho y^2 r}{m^2} f'\right)' - \left\{\frac{\rho y^2}{r} - \frac{p'}{\rho r}\left(\rho' - \frac{p'}{c_T^2}\right)\right\} f = 0, \tag{4}$$

where $c_T = \sqrt{\gamma p/\rho}$ is the velocity of sound. We find from (4) the local convective stability condition[4] $p'N' < 0$, $N \equiv p\rho^{-\gamma}$.

In the case of a thin, uniformly rotating disk $\nu_\varphi = v_\varphi / r = \text{const}$, for a polytropic dependence of the pressure on the density $p\rho^{-\gamma_0} = \text{const}$, $\gamma_0 = 3$, we can write the equilibrium functions which satisfy Eq. (2) as

$$\rho = \rho_0(1 - r^2/R^2)^{1/2}, \quad p = p_0(1 - r^2/R^2)^{3/2}, \quad \phi' = \nu_0^2 r, \quad \nu_\varphi^2 = \nu_0^2 - 3p_0/\rho_0 R^2, \tag{5}$$

where $\nu_0^2 = \pi^2 G\sigma_0/2R$, $\sigma = \rho h$ is the surface density, and R is the radius of the disk.

The equilibrium configuration (5) is convectively unstable at $\gamma < \gamma_0 = 3$. The growth rate of an instability can be estimated from Eq. (4), which can be written for the steady state (5) as follows:

$$x(\sqrt{1 - x^2}\, x f')' - (m^2\sqrt{1 - x^2} - \kappa x^2/\sqrt{1 - x^2}) f = 0, \tag{6}$$

where $x = r/R$, and $\kappa = 3m^2 c_{T_0}^2 (1 - 3/\gamma)/\gamma R^2 y^2$. Solving (6) by the variational method with the trial functions $f_m = x^m(1 - x^2)$, we find the following expression for the frequency:

$$\omega = -m\nu_\varphi \pm i\sqrt{3/2\gamma}\sqrt{(3/\gamma - 1)\Lambda_m}\, c_{T_0}/R, \tag{7}$$

where the parameter $\Lambda_m \simeq 0.3$, 0.8, and 0.5, respectively, for $m = 1, 2, 3$.

3. The complete system of equations (1) was solved numerically in the region $0 \leqslant r \leqslant 1$, $0 \leqslant \varphi \leqslant 2\pi$ with the boundary condition $v_r(R, \varphi) = 0$ under the assumption that the gravitational potential ϕ is constant. The equilibrium configuration (5) is the initial configuration. The initial velocity perturbation, which satisfies the condition $\operatorname{div} \mathbf{v} = 0$, is specified by the stream function $\psi = \lambda r^m(1 - r^2)\sin m\varphi$. The calculations were carried out for $\lambda = 0.1$, $\gamma = 5/3$, $\nu_0 = 2$, $\rho_0 = 1$, $p_0 = 1$, and $R = 1$.

Figure 1 is a plot of the kinetic energy of radial motion as a function of time $E_r = \frac{1}{2}\int \rho v_r^2 dV$ for $m = 2$ and $m = 3$. The growth rate of E_r during the development of an instability is in qualitative agreement with the results of the linear theory. Over the interval of time $\tau = \sqrt{p_0/\rho_0}\, t/R \simeq 5$, the functions $E_r(t)$ peak and the fall off.

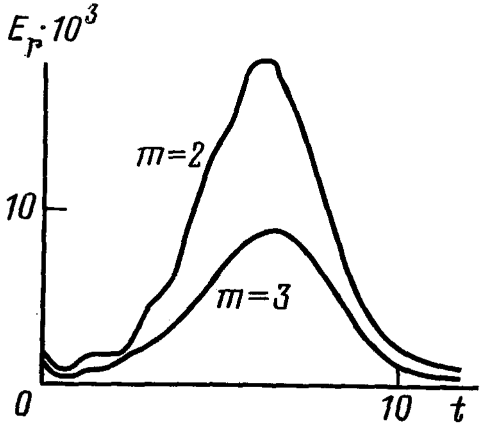

FIG. 1.

The evolution of the isentropic lines $N = \text{const}$, which were frozen in the substance, for the second mode ($m = 2$) and third mode ($m = 3$) is shown in Figs. 2 and 3, respectively. At $t = 0$, these curves are concentric circles with radii $r_1 = 0.2$, $r_2 = 0.3$ (for $m = 2$), $r_2 = 0.4$ (for $m = 3$), and $r_3 = 0.8$. We see that the high-temperature inner layers of gas transform into spiral sleeves during the evolution. The convective nature of the growth of an instability largely manifests itself in the fact that the inner layers are carried out to the surface and the outer layers are drawn into the interior. In the absence of dissipation, the topology of the lines of the constant level of the frozen-in function $N(r,\varphi)$ remains constant, which necessarily leads to the formation of narrow inversion layers. As a result of the development of an instability, the configuration, with the exception of the inversion layers, on the average reaches the stability region which is characterized by a constant entropy ($N = \text{const}$).

4. The results of our study show that the mechanism proposed here may be viewed as a model describing the formation of the spiral structure of galaxies in a pregalactic rotating gaseous disk as a result of the development of a convective instability. In contrast with the popular current conception of the "density waves,"[5–7] the

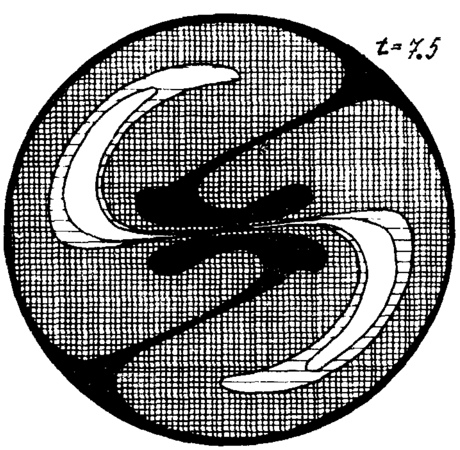

FIG. 2

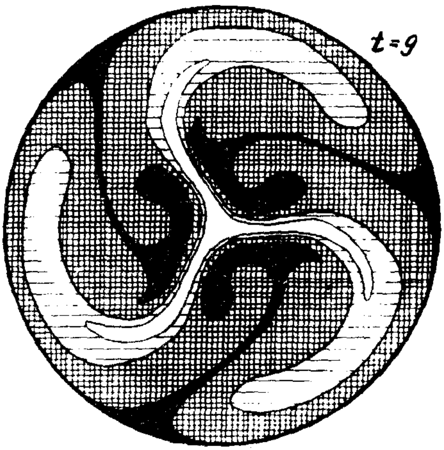

FIG. 3.

formation of a spiral structure is, in our view, a one-time development of an instability in a pregalactic gaseous cloud. The subsequent preferential formation of stars in the neighborhood of the central region, on the bridge, and in the sleeves is due to the higher concentration of the gas density and temperature in these regions. A necessary condition for the onset and rapid growth of convective instability is a rapid decrease in entropy from the center to the periphery. This condition is satisfied during the initial gravitational contraction of the gaseous disk. The rotating disk causes a twisting of the branches and the formation of a characteristic rotating spiral pattern with lagging branches. The development of an instability consumes the gravitational and thermal energy of the gaseous cloud. Having exhausted the reserves of "unstable" energy, the system undergoes a transition to a new, nearly steady state with a nearly steady entropy, with the exception of narrow inversion layers which are the prospective sleeves of the spiral galaxy.

The hypothesis proposed here is by no means a complete and detailed description of several distinctive features of the observed spiral galaxies. In explaining the origin of the spiral structure, however, this hypothesis makes use of a minimum number of assumptions. An outburst of a supernova, which can also be explained in a most natural way by the development of a *two-dimensional convective instability,* is a close analog of the mechanism considered here.[8]

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