Precession soliton in a ferromagnetic film

A. S. Kovalev, A. M. Kosevich, I. V. Manzhos, and K. V. Maslov
A. M. Gor'kiĭ State University, Khar'kov

(Submitted 28 May 1986; resubmitted 4 July 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **44**, No. 4, 174–177 (25 August 1986)

A nonlinear integro-differential equation which describes a magnetization soliton of a thin ferromagnetic film is derived. A localized solution of this equation is found by using numerical methods on a computer. The stability of a magnetic soliton is discussed.

Experimental efforts are now underway to detect magnetic solitons—specific localized excitations in magnetically ordered media.[1–3] Although the general theory of such excitations has been thoroughly developed,[4] the quantitative results have been obtained largely for the one-dimensional case, ignoring the magnetic dipole interaction. Experiments, on the other hand, have been carried out primarily with garnet-

 0021-3640/86/160222-04$01.00

ferrite film samples,[1,2] for which it is important, and in several cases essential, to take demagnetizing fields into account. It is therefore necessary to develop a theory of magnetic solitons in samples of finite dimensions. In the present letter we develop a very simple version of this theory.

We consider an easy-axis ferromagnet film of thickness h, whose easy axis lies in the plane of the film and coincides with the $0y$ axis. The $0z$ axis is directed along the normal to the film. We confine the analysis to the case in which the magnetization depends exclusively on one coordinate—the x coordinate.

In a thin ferromagnetic film under the condition $\mathbf{M} = \mathbf{M}(x)$ the magnetization dynamics is determined by the Landau-Lifshitz equation

$$\frac{\hbar}{2\mu_0}\frac{\partial \mathbf{M}}{\partial t} + \alpha\left[\mathbf{M}\frac{\partial^2 \mathbf{M}}{\partial x^2}\right] + \beta[\mathbf{M}\mathbf{n}_y]M_y + [\mathbf{M}\mathbf{H}^{(m)}] = 0, \quad (1)$$

where α and β are the exchange constant and the anisotropy constant ($\beta > 0$), respectively, and the magnetic field $\mathbf{H}^{(m)}$ is expressed in terms of the magnetization distribution

$$H_x^{(m)} = 2\pi h\frac{\partial}{\partial x}\hat{g}M_x, \qquad H_y^{(m)} = 0, \quad H_z^{(m)} = -4\pi M_z - 2\pi h\frac{\partial}{\partial x}\hat{g}M_z. \quad (2)$$

Here we use the standard notation for the Hilbert transform

$$\hat{g}u(x) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{u(x')dx'}{x'-x}. \quad (3)$$

One-dimensional equations (1)–(3) hold if the scale dimension Δ of the nonuniform distribution of the magnetization along the $0x$ axis is appreciably larger than the film thickness: $\Delta \gg h$.

Measuring the time in units of $\hbar/2\beta\mu_0$ (where M_0 is the nominal magnetization) and the coordinate in units of $l_0 = \sqrt{\alpha/\beta}$, we find the following integro-difference equation for the unit magnetization vector $\mathbf{m} = \mathbf{M}/M_0$:

$$\frac{\partial \mathbf{m}}{\partial t} + \left[\mathbf{m}\frac{\partial^2 \mathbf{m}}{\partial x^2}\right] + [\mathbf{m}\hat{J}\mathbf{m}] = 0, \qquad \hat{J} = \mathrm{diag}\left(\gamma\frac{\partial}{\partial x}\hat{g},\ 1,\ -Q-\gamma\frac{\partial}{\partial x}\hat{g}\right), \quad (4)$$

where $Q = 4\pi/\beta$, and $\gamma = Qh/2l_0$.

In a linear approximation in m_x and m_z, we find from (4) the dispersion law for the spin waves (the dependence of the frequency ω on the wave vector k)

$$\omega^2 = (1 + k^2 + \gamma|k|)(\omega_0^2 + k^2 - \gamma|k|), \qquad \omega_0^2 = 1 + Q, \quad (5)$$

which is the same dispersion law in the limit $kh/l_0 \ll 1$ as the one obtained in Refs. 5 and 6.

Let us analyze the solution of Eq. (4) which corresponds to low-amplitude solitons at rest, for which $\omega_0 - \omega \ll \omega_0$. We will use the asymptotic procedure,[7] choosing $\epsilon = \sqrt{1 - \omega/\omega_0} \ll 1$ as the small expansion parameter. In the principal approximation in ϵ we can then write

$$m_z = \epsilon\varphi(x)\cos\omega t, \qquad m_x = \epsilon\omega_0\varphi(x)\sin\omega t, \tag{6}$$

where $\varphi(x)$ is derived as a solution of the equation

$$(1 + \omega_0^2)\frac{\partial^2\varphi}{\partial x^2} + Q\gamma\hat{g}\frac{\partial\varphi}{\partial x} - \epsilon^2\omega_0^2\left\{2 - (1 + \omega_0^2)\frac{\varphi^2}{2}\right\}\varphi = 0. \tag{7}$$

For $\partial\varphi/\partial x \ll \varphi\gamma Q/(2+Q)$ the last term in (7) may be dropped, which corresponds to the so-called exchangeless approximation. In this approximation we have

$$\varphi(x) = \frac{2}{\sqrt{1+\omega_0^2}} f(\xi), \qquad \xi = \frac{2\omega_0^2}{Q\gamma}\epsilon^2 x, \tag{8}$$

where the function $f(\xi)$ satisfies the equation

$$f - f^3 - \frac{\partial}{\partial\xi}\hat{g}f = 0. \tag{9}$$

The soliton solution of Eq. (9) is found numerically on a computer. The function $f = f(\xi)$ is represented by curve 1 in Fig. 1. Curve 2 in Fig. 1 shows, for comparison, a self-similar solution of the Benjamin-Ono equation[8] which corresponds to the exchange $f^3 \to f^2$ in Eq. (9).

The major features of this soliton are the nonexponential (power-law) asymptotic behavior of this solution at large spatial separations $[f(\xi) \sim \xi^{-2}$ as $\xi \to \pm\infty]$ and an unusual (for low-amplitude solitons) dependence of the localization region Δ on the amplitude ϵ ($\Delta \propto \epsilon^{-2}$: the soliton is localized only slightly). Expression (8) implies that the inequality $\Delta \gg h$ is satisfied if $1 - \omega/\omega_0 \ll Q^2/4\omega_0^2$, and the inequality $\partial\varphi/\partial x \ll \varphi Q\gamma/(2+Q)$ is satisfied if $1 - \omega/\omega_0 \ll (h/l_0)^2 Q^4/8\omega_0^2(2+Q)$. Which of these two inequalities is the more rigorous one is determined by the parameter $hQ/l_0\sqrt{2+Q}$, which can theoretically have any value with respect to unity. If, however, ω is close enough to ω_0, then both inequalities can be satisfied, assuring the validity of this approximation.

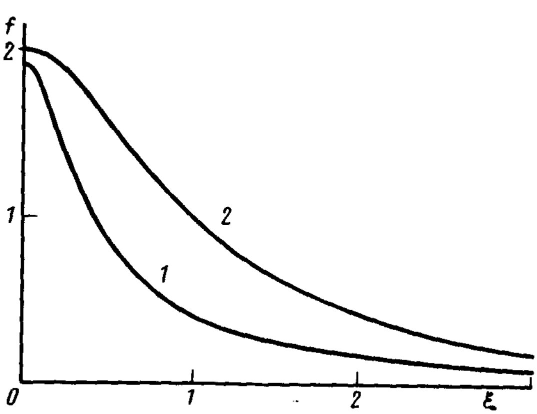

FIG. 1.

Let us determine the number of magnons N bound in the soliton. In the semiclassical approximation, we must calculate in this case the adiabatic invariant[4] $I = N/\hbar$. Computer calculations have shown that $\int_{-\infty}^{\infty} d\xi f^2(\xi) \approx 2.49$, so that the number of bound magnons per unit length along the $0y$ axis in the limit $\omega \to \omega_0$ is

$$N(\omega_0) \approx 0.62(M_0/\mu_0)h^2Q^2/\omega_0(1 + \omega_0^2). \tag{10}$$

In the limit $\omega \to \omega_0$, the presence of a limiting value of N resembles the situation for the dynamic two-dimensional solitons.[4] In our case, however, expression (10) contains the factor Q^2, and the nonzero limiting value of N is attributable exclusively to the fact that the magnetic-dipole interaction is taken into account. The threshold value of $N(\omega_0)$ seems to complicate the experimental problem of excitation of magnetic solitons in films.

To clarify the problem of the stability of these solitons, we must know the sign of the derivative $dN/d\omega$ in the limit $\omega \to \omega_0$. This sign can be determined if the solution for a low-amplitude soliton is known even for a single frequency near ω_0. Equation (7) is consistent with an exact solution

$$\begin{Bmatrix} m_x \\ m_z \end{Bmatrix} = \frac{4\gamma Q/3(1+\omega_0^2)\omega_0}{1 + [\gamma Qx/3(1+\omega_0^2)]^2} \begin{Bmatrix} \omega_0 \sin \omega_* t \\ \cos \omega_* t \end{Bmatrix} \tag{11}$$

for a particular frequency

$$\omega_* = \omega_0 \{1 - \gamma^2 Q^2 / 6\omega_0^2(1 + \omega_0^2)\} < \omega_0 . \tag{12}$$

Semiclassical quantization of solution (11) gives the number of magnons bound in the soliton

$$N(\omega_*) \approx \frac{\pi}{3}(M_0/\mu_0)h^2Q^2 / \omega_0(1 + \omega_0^2) > N(\omega_0). \tag{13}$$

Consequently, we have $dN/d\omega < 0$ near the frequency ω_0. In all problems considered previously[4] this sign of inequality showed that dynamic solitons are stable.

We wish to thank A. V. Tartakovskiĭ for assistance in solving Eq. (9) on a computer.

[1]B. A. Kalinikos *et al.* Pis'ma Zh. Eksp. Teor. Fiz. **38**, 343 (1983) [JETP Lett. **38**, 413 (1983)].
[2]V. S. Gornakov, L. M. Dedukh, and V. N. Nikitenko, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 199 (1984) [JETP Lett. **39**, 236 (1984)].
[3]A. I. Smirnov, Zh. Eksp. Teor. Fiz. **88**, 1369 (1985) [Sov. Phys. JETP **61**, 815 (1985)].
[4]A. M. Kosevich, B. A. Ivanov, and A. S. Kovalev, Nelineĭnya volny namagnichennosti (Nonlinear Magnetization Waves), Naukova dumka, Kiev, 1983.
[5]B. A. Kalinikos, Izvestiya Vyssh. uch. zav, ser. fizika **8**, 42 (1981).
[6]R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids **19**, 308 (1961).
[7]A. M. Kosevich and A. S. Kovalev, Zh. Eksp. Teor. Fiz. **67**, 1793 (1974) [Sov. Phys. JETP **40**, 891 (1974)].
[8]R. I. Joseph, J. Math. Phys. **18**, 2251 (1977).

Translated by S. J. Amoretty