Condition of asymptotic freedom and temperature of the relict radiation of the universe

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If the universe has passed through a quantum era of expansion, which was characterized by a Planck temperature $\Theta_p \sim 10^{32}$ K, then the present-day temperature of the relict radiation is close to the observed one, provided that the equation of state at ultrahigh density is of the form $P = (1/3)\,\rho c^2$. This is a strong argument in favor of the hypothesis of asymptotic freedom (exclusion of the interaction of hadrons at short distances). It is also indicated that the Planck length $l_p \sim 10^{-33}$ cm is empirically singled out in comparison with other possible values of the fundamental length.

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1. An essential feature of modern theory of strong interactions—quantum chromodynamics[1]—is the notion of asymptotic freedom, i.e., the exclusion of hadron interactions at short distances. In the picture wherein the hadrons are regarded as quarks bound by vector gluons, this exclusion of the interaction means that at ultrahigh densities matter consists of free quarks and satisfies the equation of state[2]

$$P \to \frac{1}{3}\rho c^2. \tag{1}$$

In this article we wish to point out a curious possibility of verifying the asymptotic-freedom hypothesis at ultrahigh densities by using the dependence of the rate of cosmological expansion on the equation of state. As the quantity whose present-day value depends significantly on the rate of the cosmological expansion (meaning also on the equation of state) we choose the temperature of the relict radiation of the universe.

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2. We start from the assumption (which is now shared by many cosmologists) that the universe has passed through a quantum era of expansion characterized by a density $\rho_p = c^5/G^2\hbar = 5\times 10^{93}$ g/cm^3. In this case it is quite natural to assume that the initial temperature of the universe had the Planck value[3]

$$\Theta_p = \frac{1}{k}(\hbar c^5/G)^{1/2} = 1.4\cdot 10^{32}\text{K}. \tag{2}$$

This value of the temperature in the quantum era followed not only from dimensionality considerations, but also from the analogy between the processes of particle production by a black hole and cosmological particle production.[4]

Recognizing that the quantum production of particles takes place most intensively at $t\sim t_p = (G\hbar/c^5)^{1/2} = 5\times 10^{-44}$ sec, it is natural to neglect this production (including also the production of massless particles—photons) during later stages of the expansion.

Let us estimate the present-day radiation temperature $\Theta(t_0)$, which has cooled from $\Theta(t_p) = \Theta_p$ in accordance with the rate of the cosmological expansion

$$\Theta(t_o) = \Theta_p a(t_p)/a(t_o), \tag{3}$$

where a is a scale factor. For the equation of state $P=\gamma\rho c^2$ we have in the quasi-Euclidean cosmological model $a\sim t^\Gamma$, where $\Gamma = 2/3(\gamma+1)$.

We assume for simplicity a "piecewise-constant" equation of state, breaking up the dynamic history of the universe into three epochs:

(t_p, t_1) $\gamma=\gamma_1$ superdense state of matter,
(t_1, t_{eq}) $\gamma=1/3$ gas of ultrarelativistic particles,
(t_{eq}, t_0) $\gamma=0$ dustlike matter.

Here $t_{eq} = 3\times 10^3$ years is the instant when the density of matter is equal to the radiation (the total density of matter of the universe is assumed for simplicity to be equal to the critical density of the Friedmann models), $t_0 = 5\times 10^{17}$ sec is the present-day age of the universe.

Taking the foregoing breakdown into account, we find, in accordance with (3), the present temperature of the relict radiation

$$\Theta(t_o) = t_p^{\Gamma_1} t_1^{1/2-\Gamma_1} t_{eq}^{1/6} t_o^{-2/3}\Theta_p. \tag{4}$$

The numerical value of $\Theta(t_0)$ depends essentially on the equation of state at ultrahigh densities. If the asymptotic-freedom condition is satisfied as $\rho\to\rho_p$, then according to (1) we have $\gamma_1 = 1/3$ and

$$\Theta(t_o) = t_p^{1/2} t_{eq}^{1/6} t_o^{-2/3}\Theta_p = 1\text{K}, \tag{5}$$

which is close to the observed temperature 3 K of the relict radiation. At the same time, the extremely stringent equation of state with $\gamma=1$ at $\rho_p \gtrsim \rho \gtrsim 10^{50}$ g/cm^3,[5] which corresponds to a value $t_1 \approx 10^{-22}$ sec, would lead to a present-day relict-radiation temperature larger by a factor $(t_1/t_p)^{1/6} = 4\times 10^3$ than the value (5) corresponding to the condition of asymptotic freedom, when $\gamma = 1/3$.

Thus, the observed temperature of the relict radiation can serve as a serious argument in favor of the condition of asymptotic freedom at ultrahigh densities.

3. The described approach can be applied to the problem of the fundamental length (this was pointed out to us by V. L. Ginzburg). The existence of a fundamental length l_f would mean also the presence of a fundamental (maximal) temperature $\Theta_f = \hbar c / k l_f$ and a fundamental density $\rho_f = \hbar / c l_f^4$.[6] Assume that at the cosmological instant of time corresponding to the density ρ_f the temperature was equal precisely to Θ_f. Then the asymptotic-freedom hypothesis, i.e., the assumption of a "soft" equation of state $P = \rho c^2/3$ up to the density ρ_f, leads to the same present-day relict-radiation temperature as in (5), where the role of l_f is assumed by the Planck length $l_p = (\hbar G/c^3)^{1/2} = 2\times10^{-33}$ cm. The reason is that in a cosmological model that "starts" with $\rho = \rho_p$ and $\Theta = \Theta_p$ and satisfies an equation of state with precisely $\gamma = 1/3$ during a certain initial interval of time, the density and temperature in this interval are connected by the relation $\rho(t) = k^4\Theta^4(t)/c^5\hbar^3$, i.e., the same relation that should exist between the fundamental density and the fundamental temperature. (Incidentally, it is natural to require that the last relation coincide identically with the Stefan-Boltzmann law; then the relation between the fundamental density and the temperature is determined by the density, up to the dimensionless coefficient).

In this connection, the restrictions on the initial equation of state remain in force: any "hardening" of the equation of state would lead to an increase of the relict-radiation temperature. From this we can in turn obtain a limitation on the measure of the hardness of the equation of state near ρ_f (for example, a limitation on the duration of the action of the equation of state with $\gamma = 1$, or a limitation on the constant b in the equation of state used in[7]).

4. However, the value of the fundamental length $l = l_p$ is singled out for the following reason. Replacing in (4) the subscript "p" by "f" (and assuming for simplicity $\gamma_1 = 1/3$), we obtain

$$\Theta(t_0) = t_f^{1/2}\, t_{eq}^{1/6}\, t_0^{-2/3}\, \Theta_f .$$

If we take the instant t_f to be not the instant when the density ρ_f is reached, but put in "natural" manner $t_f = l_f/t$, then $\Theta(t_0) \sim 10^{-7}(l_f/10^{-17}\text{ cm})^{-1/2}$ K, which coincides with the observed temperature only if $l_f = l_p$. A possible difference between the density of matter from critical has little effect on the value of $\Theta(t_0)$.[4] Thus, among the possible values of l_f, only the Planck length is singled out from the empirical-cosmological point of view.

5. Summarizing, we can conclude that the cosmological arguments advanced in the present article, first, favor the asymptotic freedom for ultrahigh densities and, second, offer evidence that the Planck length occupies, from the empirical point of view, a preferred position relative to other possible values of the fundamental length.

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