Annihilation widths and shifts of quasinuclear $N\bar{N}$ levels in the coupled-channel model

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It is shown that in the case of radius of the annihilation interaction, the annihilation widths and shifts of the quasinuclear levels $(N\bar{N})$ remain small at arbitrary annihilation intensity. At low annihilation intensity the level shift is negative—the level becomes deeper.

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Theoreticians pay more attention presently to systems that are maintained in a bound state by an interaction with a large effective radius of the forces, but are made unstable by short-range annihilation interaction. Examples are bound states of a nucleon and an antinucleon (quasinuclear mesons[1]), for which the annihilation-interaction radius $r_a \sim (2m_N)^{-1} \sim 0.1$ F is much less than the characteristic dimension of the $(N\bar{N})$ system (1 F).

The bound-state spectrum of these systems, neglecting the annihilation direction, is obtained by solving the Schrödinger equation with a real potential. Allowance for the annihilation interaction causes the levels to shift and to acquire a width. Attempts were recently made to calculate the shifts and widths of the quasinuclear mesons with the aid of the complex optical-model potential.[2] However, as shown in[3], the optical potential can be used only in the scattering problem and is not suitable for the solution of an eigenvalue problem. Correct allowance for annihilation is possible only via a multichannel analysis, and leads to results that differ in principle from the predictions of the optical model.

In the present paper we consider the problem of the annihilation shift and width of a level in a nonrelativistic model of two coupled channels. Each of the channels is assumed to be of the two-particle type: channel h contains two heavy particles with mass m (the analog of the $N\bar{N}$ channel), and the channel l corresponds to two interacting particles with mass $\mu < m$ (the analog of the boson annihilation channel). We assume that all particles have zero spin and are nonrelativistic—the last condition is actually satisfied, for example, for the processes $N\bar{N}\to 2\rho$ and $N\bar{N}\to 2\omega$.

The Hamiltonian $\hat{H}$ in the Schrödinger equation $\hat{H}\Psi = E\Psi$ is described by the Hermitian 2×2 matrix

$$\hat{H} = \begin{pmatrix} \hat{H}_l & \hat{H}_{lh} \\ \hat{H}_{hl} & \hat{H}_h \end{pmatrix} \tag{1}$$

where the diagonal elements $\hat{H}_l$ and $\hat{H}_h$ are the Hamiltonians of the channels l and h and are respectively equal to $\hat{H}_1 = k^2/\mu - 2(m-\mu)$ and $\hat{H}_h = p^2/m + V(r)$, while the off-diagonal elements $\hat{H}_{lh} = \hat{H}_{hl}$ couple the channels with each other (the last equality is true because the Hamiltonian H is Hermitian and T-invariant). The wave function Ψ has two components Ψ_l and Ψ_h, corresponding to the channels l and h.

The Schrödinger equation can be easily investigated by choosing $\hat{H}_{lh}$ in separable form (for simplicity we confine ourselves to a partial wave with $l=0$)

$$\hat{H}_{lh} = g\sqrt{m\mu}\,\xi(r)\xi(r'), \tag{2}$$

where g is a dimensional constant and

$$\xi(r) = \sqrt{\frac{2}{r_a}}\exp\left(-\frac{r}{r_a}\right)\Big/r,$$

with $\int \xi^2(r)r^2\,dr=1$. The quantity r_a determines the radius of the annihilation interaction.

We rewrite the Schrödinger equation in the form

$$\left.\begin{aligned} |\Psi_l\rangle &= g\sqrt{m\mu}\,(E-H_l)^{-1}|\xi\rangle\langle\xi|\Psi_h\rangle \\ |\Psi_h\rangle &= g\sqrt{m\mu}\,(E-H_h)^{-1}|\xi\rangle\langle\xi|\Psi_l\rangle \end{aligned}\right\} \tag{3}$$

After multiplying (3) from the left by $\langle\xi|$, the unknown functions $|\Psi_l\rangle$ and $|\Psi_h\rangle$ can be eliminated, and we arrive at an equation for the eigenvalues of the energy E:

$$\langle\xi|(E-H_h)^{-1}|\xi\rangle\langle\xi|(E-H_l)^{-1}|\xi\rangle = 1/g^2 m\mu. \tag{4}$$

The matrix element $\langle\xi|(E-H_h)^{-1}|\xi\rangle$ can be easily calculated by using the spectral representation of the Green's function of the heavy particles:

$$(E-H_h)^{-1} = \sum_\lambda \frac{|\phi_\lambda\rangle\langle\phi_\lambda|}{E-E_\lambda} + \int \frac{|\phi_{\mathbf{k}}\rangle\langle\phi_{\mathbf{k}}|}{E-k^2/m+i0}\,d\mathbf{k}, \tag{5}$$

where ϕ_λ and $\phi_{\mathbf{k}}$ are the wave functions of the bound states (with energy E_λ) and of the continuous spectrum of the heavy particles in the absence of a coupling between the channels. It is convenient to separate explicitly in the matrix elements $\langle\xi|\phi_\lambda\rangle$ the factor $(r_a/R)^{3/2}$, where R characterizes the dimension of the system in the state λ:

$$\langle\xi|\phi_\lambda\rangle = \int \xi(r)\phi_\lambda(r)r^2\,dr = (r_a/R)^{3/2}a_\lambda \tag{6}$$

so that the parameters a_λ turn out to be of the order of the energy. Equation (4) reduces in this notation to the form

$$\sum_\lambda \frac{a_\lambda^2}{E-E_\lambda} = \left(\frac{R}{r_a}\right)^3 \frac{[1-iK(E)r_a]^2}{g^2 m\mu^2 r_a^2} + \frac{m r_a^2}{[1-i\kappa(E)r_a]^2} \tag{7}$$

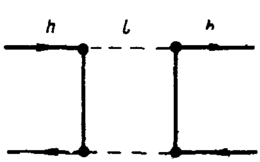

FIG. 1.

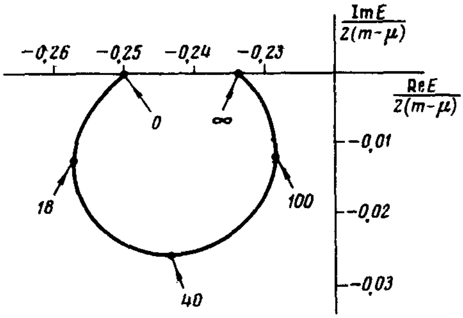

FIG. 2.

Here $K(E)=[\mu(E+2m-2\mu)]^{1/2}$ and $\kappa(E)=(mE)^{1/2}$. The derivation of (7), the functions $\phi_{\mathbf{k}}$ of the continuous spectrum were replaced by plane waves, a procedure permissible if r_a is small enough and $V(r)$ does not have an excessively singular behavior as $r\to 0$.

In the case of a weak coupling between the channels, when $|g|m\mu r_a^2 \ll 1$, the level shift ΔE and the width Γ can be obtained by perturbation theory

$$\Delta E \equiv \mathrm{Re}\,E - E_\lambda = -(r_a/R)^3 g^2 m\mu^2 r_a^2 a_\lambda^2 \frac{1-(K(E_\lambda)r_a)^2}{1+(K(E_\lambda)r_a)^2}, \tag{8}$$

$$\Gamma = -2\mathrm{Im}E = 4(r_a/R)^3 g^2 m\mu^2 r_a^2 a_\lambda^2[1+(K(E_\lambda)r_a)^2]^{-2}. \tag{9}$$

It is easy to verify that according to (8) we have $\Delta E<0$ if $r_a \lesssim \sqrt{2}/m$. Thus, under known conditions, the annihilation increases the binding energy λ (the level becomes deeper). The physical reason for this phenomenon is quite simple, namely, the coupling with the channel l can be considered in the channel h as an additional interaction $V_h=H_{hl}(E-H_l)^{-1}H_{lh}$. In the region where perturbation theory is valid, the sign of the annihilation shift ΔE is determined by the sign of $\mathrm{Re}V_h$. In a realistic quantum field-theoretical model the interaction V_h corresponds, apart from the sign, to the Feynman diagram of Fig. 1. It is easy to verify that the real part of this diagram is positive-definite, and consequently $\Delta E<0$ (the diagram of Fig. 1 corresponds to $r_a=1/2m$).

Another result of our analysis is the demonstration of the fact that the annihilation shifts and widths are small if the parameter r_a/R is small. Outside the range of perturbation theory, Eq. (7) can be solved numerically. Figure 2 shows motion of the level in the channel each with increasing constant $g^2(r_a/R=0.1, \alpha=1)$. As seen from the figures, the annihilation shifts remain small at low values of g^2. For the case $N\bar{N}\to 2p$ (the binding energy at $g=0$ is $\epsilon=85$ MeV), the maximum shift is $\Delta E=15$ MeV ($g^2\approx 100$) and the maximum width is $\Gamma\approx 5$ MeV (at $g^2\approx 50$). Comparison of Fig. 2 with the predictions of the optical model for the $N\bar{N}$ system[2] shows that the multichannel approach that corresponds to the physical nature of the phenomenon has nothing in common with the use of the optical model for the calculation of the annihilation shifts at the levels of the quasinuclear (NN). The fact that the Hamiltonian of the optical model is not Hermitian always leads to a "pushing-out" of the level, and the annihilation shift and width increase without limit with increasing imaginary part of the potential (regardless of its radius). On the other hand, the true picture of the level motion as a result of annihilation effects is much more complicated, as shown above. The principal fact is that the smallness of the dimensions of the annihilation region in comparison with the radius of the orbit of the finite motion ensures smallness of annihilation shifts and widths.

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