Connection between the renormalizability of the theory and the behavior of the transport coefficients

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The temperature dependence of the transport coefficients of an ultrarelativistic gas of elementary particles is discussed as a function of the type of interaction.

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Allowance for dissipative processes in elementary-particle systems whose temperature and density are high has recently attracted interest. These processes seem to play an important role in multiple production of particles[1] as well as in astrophysics and cosmology.[2] It becomes important in these cases to determine the temperature dependences of the transport coefficients. It is known that for interactions with a dimensionless coupling constant this dependence can be obtained in the ultrarelativistic region from dimensionality considerations.[3] In the present paper, analogous estimates are obtained for an arbitrary local polynomial interaction with one coupling constant. This makes it possible, in particular, to obtain the connection between the behavior of the transport coefficients at high temperatures and the renormalizability of the theory.

Let us determine, for example, the behavior of the shear-viscosity constant η on the type of interaction. Our purpose being the determination of the temperature dependence of the viscosity in the ultrarelativistic region $T \gg m$, it is convenient to choose in the system of units $c=\hbar=k_B=1$ (k_B is the Boltzmann constant) the degree as the principal unit. Then, for different polynomial interactions, the dimensionality of the coupling constant is $[g]=[T]^{-2n}$ ($n=0,1,2,\cdots$), where n characterizes the degree of nonlinearity. We have excluded here odd powers because of the instability of the corresponding theories. At $n=0$ the interaction belongs to the renormalizable class (R), and in the remaining cases it is nonrenormalizable (N). The coupling constant will be assumed to be quite small.

From symmetry considerations we can estimate the correlation length in the system $l_{\text{corr}} \sim g^{-2} T^{-1-4n}$. The assumption that the coupling constant is small ensures simultaneous satisfaction of the conditions of applicability of the quantum relativistic kinetic theory $l_{\text{corr}} \gg \lambda \sim T^{-1}$ (λ is the de Broglie wavelength) and hydrodynamics $l_{\text{corr}} \ll L$ (L is the characteristic scale of the macroscopic process) i.e., $\lambda \ll l_{\text{corr}} \ll L$. In second-order perturbation theory we have for quasi-homogeneous systems, just as in the nonrelativistic kinetic theory, $\eta \sim g^{-2}$ (e.g.,[4]), since by definition $[\eta]=[T]^3$, we obtain

$$\eta \sim g^{-2} T^{3-4n}. \tag{1}$$

For example, for interactions between scalar particles of the type

$$L_a = g\phi^{2n+4}, \quad L_b = g(\phi_{,a}\phi^{,a})^{n+1}, \tag{2}$$

we obtain respectively

$$\eta_a \sim g^{-2} T^{3-4n}, \quad \eta_b \sim g^{-2} T^{3-8n}. \tag{3}$$

In particular, the known result $\eta \sim T^3$ of Izo, Mori, and Namiki[5] for the viscosity of a pion as corresponds in our analysis to the renormalizable $g\phi^4$ model. The same temperature dependence $\eta \sim T^3$ holds also in quantum electrodynamics. For the viscosity of a neutrino gas in the four-fermion weak-interaction model we obtain $\eta \sim T^{-1}$, which agrees with the results of[6]. On the other hand, $n = 0$ in the unified theory of weak and electromagnetic interactions, so that the viscosity of a neutrino gas in this theory is $\eta \sim T^3$.

We note that when the dimensionality of space changes the temperature dependence of η changes for all types of interactions, with the exception of quantum electrodynamics and of theories of analogous structure.

In the general case it turns out that η increases or decreases with temperature, depending on whether the type of interaction is of class R or class N. This is valid also for such transport coefficients as the thermal conductivity and the bulk viscosity. This qualitatively different behavior of the transport coefficients gives ground for hoping that comparison with experiments will make it possible to identify the class to which the interaction in the corresponding processes belongs.[1)]

It is known, for example, that the deductions of the Landau hydrodynamic model are in good agreement with the experimental data on multiple particle production.[1] This means that even when dissipative processes are taken into account, the hadron cluster can be regarded as an ideal liquid ($\eta \approx 0$), at least prior to the instant of the cluster breakup. On the other hand, the cluster temperature decreases in the course of its evolution. But a simultaneous decrease of the viscosity and the temperature in the course of the cluster expansion can be realized only in theories of class R.

We now obtain the dependence of η on the speed of sound c_s. For the interactions (2), the values of c_s as functions of n are known[8]

$$c_{sa}^2 = \frac{n+1}{n+3}, \qquad c_{sb}^2 = \frac{1}{2n+1}. \tag{4}$$

Then, using (3), we can obtain

$$\eta_a \sim g^{-2} T^{3-4(1-3c_s^2)/(c_s^2-1)},$$
$$\eta_b \sim g^{-2} T^{3-4(1-c_s^2)/c_s^2}. \tag{5}$$

It is easily seen that the result $\eta \sim T^{1/c_s^2}$ of[9] is valid only for renormalizable interactions, when $[g] = 0$ and $c_s^2 = 1/3$.

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[1)]In this respect, the distinguishing role of renormalizable interactions was first noted by Ezawa *et al.*,[7] who studied the connection between the equation of state and the type of interaction.

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