Explanation of a new class of polarization echo

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An explanation is offered for the experimental regularities of a new type of polarization echo in piezosemiconductors [Shiren and Melcher, Phys. Rev. Lett. **34**, 731 (1975)]. It is indicated that it is possible to observe the "conversion" echo, as well as the maxima of the echo in the case of alternating drift at low subharmonics of the initial signal.

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It is known[2,3] that polarization echo in piezoelectrics is the result of an interaction of a sound wave $(\omega_\alpha, \mathbf{q}_\alpha)$ excited at the instant $t_1=0$ by an electric field at a frequency ω_1, $(\omega_1=\omega_\alpha\equiv q_\alpha v_\alpha, \mathbf{q}_\alpha$ is the wave vector and α is the sound polarization index) and an electric field at a frequency $\Omega=2\omega_1$ (at $t_2=\tau$). This echo is observed in the backward wave $(-\omega_\alpha, \mathbf{q}_\alpha)$ at the instant of time 2τ (parametric echo), or else at the instant $T+\tau$ (holographic echo), when a third pulse (at $t_3=T$) is used to read the static hologram $(0, \mathbf{q}_\alpha)$ produced by the forward wave and by the second pulse at the frequency $\Omega=\omega_1$. The recently observed[1] new class of polarization echo in piezosemiconductors ($\sigma\sim 10^{-7}\ \Omega^{-1}\,\mathrm{cm}^{-1}$, $T=4.2$ K upon excitation with an electric field at the subharmonic $\Omega=2\omega_1/m$ $(m=1,2,\cdots)$ has not yet received a satisfactory interpretation.[1)] The purpose of the present article is to explain the experiments of[1] within the framework of a linear theory of acoustic resonance in a piezosemiconductor with an alternating electric field[4], and furthermore to indicate that an echo appears under conditions different from those of[1].

Assume that in a semiconducting plate (Fig. 1) there propagates at an angle θ to the normal an acoustic wave excited by the first pulse of an electric field at a frequency ω_1,

$$\mathbf{u}=\mathbf{b}_\alpha u_\alpha(\mathbf{r},t)\exp\{iq_\alpha x-i\omega_1 t\}+\text{c.c.}, \tag{1}$$

where $u_\alpha(\mathbf{r},t)$ is a slowly varying amplitude, $\mathbf{x}=x\mathbf{q}_\alpha/q_\alpha$, and $\mathbf{b}_\alpha$ is the polarization vector. When a second pulse $\mathcal{E}_2(t)=\mathbf{E}_0+\mathbf{E}_2\cos\Omega t$ is turned on, the sound wave (1) together with the accompanying piezoelectric field wave $\mathbf{E}_\sim$ and the electron-density wave $n_\sim$ turn out to be in the region of the homogeneous field $\mathcal{E}_2(t)$. For a semiconductor plasma, the electric field (unlike in a dielec-

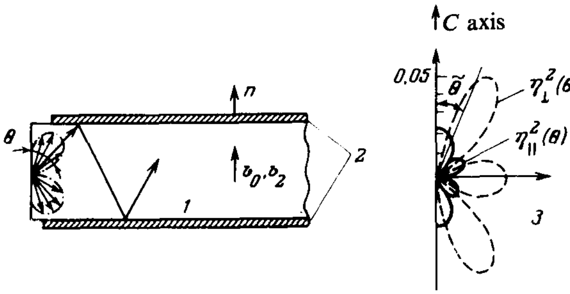

FIG. 1. Experimental setups[1]: 1—CdS plate with C axis along the normal $\mathbf{n}$,2—electrodes, 3—piezoelectric constants.[5]

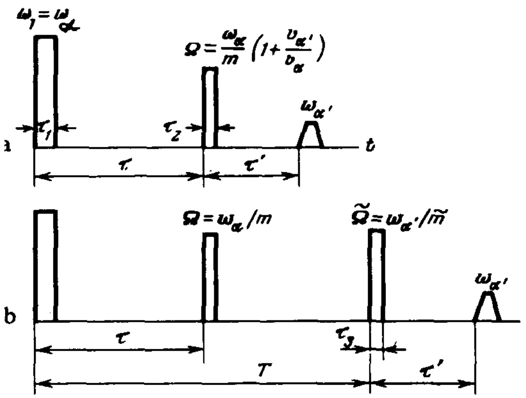

FIG. 2. Patterns of pulses ($\tau_{1,2,3}\ll\tau, v_\alpha<v_{\alpha'}$): a—parametric echo, b—holographic echo.

tric[2,3]) is not a small parameter: the alternating field excites a series of harmonics of the electron-density wave, and correspondingly harmonics of the piezoelectric field.[4] Therefore the elastic system is under conditions of parametric resonance, when the sound wave (1) becomes coupled to a backwardwave $u_{-\alpha'}(\mathbf{r},t)\exp\{iqx+i\omega_{\alpha'}t\}$ with the same wave vector $\mathbf{q}_{\alpha'}=\mathbf{q}_\alpha=\mathbf{q}$ and with frequency[4]

$$\omega_{\alpha'}=m\Omega-\omega_\alpha>0 \qquad m=1,2,\cdots. \tag{2}$$

For such a conversion of the acoustic wave α into the backward wave α' through the plasma, the propagation direction q should be piezoactive: $\eta_{\alpha'}\eta_\alpha\neq 0$, where η_α is the piezoelectric constant. For example, in CdS we have $(\gamma_\parallel\eta_\perp)_{\max}=0.026$ at $\tilde\theta=20^\circ$ for the longitudinal and the transverse waves[5] (see Fig. 1).

From the system of the continuity equation with linearized current and the Poisson and elasticity-theory equations[4] we readily obtain the equations that connect the amplitudes u_α and $u_{-\alpha'}$:

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial \mathbf{x}}\mathbf{v}_g^{\alpha\,(-\alpha')}+\Gamma_{\alpha\,(-\alpha')}\right)u_{\alpha\,(-\alpha')}=\omega_{\alpha\,(-\alpha')}R_{\alpha\,(-\alpha')}\,u_{-\alpha'\,(\alpha)}, \qquad (\omega_{-\alpha'}\equiv-\omega_{\alpha'}). \tag{3}$$

Here $\Gamma=\gamma_{\rm vis}+\gamma$, $\mathbf{v}_g$ and $\gamma_{\rm vis}$ are the group velocity and the damping decrement of the sound in the absence of carriers, while γ and R are the electronic damping rate and electronic coupling parameter. For pulses much narrower than the gap between them, $\tau_{1,2}\ll\tau$ [Fig. 2(a)], we get from (3)

$$u_{-\alpha'}(x=0,t=\tau+\tau')\approx\omega_{-\alpha'}R_{-\alpha'}(\mathbf{E}_2,\mathbf{E}_0)u_\alpha(t=0)e^{-\Gamma^0_\alpha\tau-\Gamma^0_{\alpha'}\tau'}$$

$$\times\frac{(e^{-\Gamma_\alpha\tau_2}-e^{-\Gamma_{-\alpha'}\tau_2})}{\Gamma_\alpha-\Gamma_{-\alpha'}}, \qquad \tau'=\tau\frac{v_\alpha}{v_{\alpha'}}, \tag{4}$$

(Γ^0 is the damping rate in the absence of the field), neglecting the reaction on the forward wave at the instant of action of the short second pulse:

$$|u_{-\alpha'}\ (\Delta t=(0,\tau_2))/u_\alpha(t=0)|\ll 1.$$

which is usually satisfied in experiment.[1-3] Thus, when the backward wave α' is excited, the "conversion" signal echo, which is proportional to $u_{-\alpha'}(0,\tau+\tau')$ on account of the piezoeffect, is

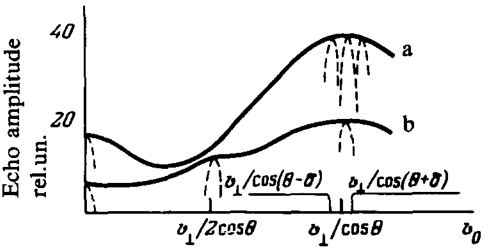

FIG. 3. Dependence of the parametric echo on the constant carrier drifts: solid curves—experiment,[1], dashed—calculation by formulas (4)–(7) for transverse waves with $\theta=30°$, where $\eta_1^2(30°)_{\max}=0.056$[5] ($\delta=10°$). a—$\Omega=\omega_1=2\pi\times 230$ MHz ($m=2$), b—$\Omega=\omega_1/2=2\pi\times 225$ MHz ($m=4$); ($v_2/v_\perp=0.3, \Gamma_M\tau_2=2$).

produced at a frequency $\omega_{\alpha'}$ at a time $\tau+\tau'$ following the turning on of the first pulse [Fig. 2(a)]. At $\Omega=2\omega_\alpha/m$. i.e., when a backward wave of the same polarization is excited, the echo appears after an interval 2τ.[1]

If the frequency of the second pulse is equal to $\Omega=\omega_\alpha/m$, then a crystal with recombination levels[6] "remembers" the mth harmonic of the nonequilibrium carrier density, $n_m(\mathbf{q}) \times \exp\{iqx-(t-\tau)/T_1\}$, which relaxes slowly with a time T_1. Such a hologram can be "read" by a third field pulse at a frequency $\tilde{\Omega}=\omega_{\alpha'}/\tilde{m}$ at the instant $t=T(T<T_1)$. The echo arises at a frequency $\omega_{\alpha'}$ at a time $T+\tau'$ after the first pulse [Fig. 2(b)]. At $\tilde{\Omega}=\omega_\alpha/\tilde{m}$ the echo arises after a time $T+\tau(\tilde{\Omega}=\omega_\alpha$ in[1]).

The greatest influence on the elastic system is exerted by the plasma when one of the harmonics (l) of the density N is in the drift-resonance region[4]:

$$\Delta_l \ll \Omega(\Delta_l=\omega_\alpha-\mathbf{q}\mathbf{v}_0-l\Omega,\ l=0,\pm 1,\cdots),$$

$$\text{i.e., } v_0\approx v_{l,m}(\theta)=\frac{v_\alpha}{\cos\theta}\left(1-\frac{l}{m}\left(1+\frac{v_{\alpha'}}{v_\alpha}\right)\right) \tag{5}$$

$$\text{at } 2\pi a/\Omega \ll 1\left(a=\omega_\alpha\left(\frac{\omega_c}{\omega_\alpha}\right)+\frac{\omega_\alpha}{\omega_D}\right),\ \Omega=\omega_\alpha\left(1+\frac{v_{\alpha'}}{u_\alpha}\right)\Big/m \quad \text{from (2)} \tag{6}$$

Here $\omega_D=v_\alpha^2/\omega_c r_0^2$, ω_c is the Maxwell frequency, r_0 is the Debye radius, $\mathbf{v}_0=\mu\mathbf{E}_0$, μ is the mobility. We consider below only this case, inasmuch as in[1] the condition (6) was satisfied: $\omega_c\sim 10^6$ sec^{-1}, $\omega_\alpha\sim 10^9$ sec^{-1}, $\omega_D\sim 10^{12}$ sec^{-1}, and the parameters $\gamma(\gamma\gg\gamma_{\text{vis}}$ and R turn out to be approximately a/Ω times larger than at $v_0\neq v_{l,m}(\theta)$ and are of the simple form (cf.[4]):

$$\gamma_{\substack{\alpha\\(-\alpha')}}=\tfrac{1}{2}\eta^2_{\substack{\alpha\\(\alpha')}}\,\omega_{\substack{\alpha\\(-\alpha')}}\frac{\omega_c\Delta_{l(l-m)}}{\Delta^2_{l(l-m)}+a^2}J^2_{l(l-m)}(\xi);$$

$$R_{\substack{\alpha\\(-\alpha')}}=\frac{(-1)^m}{2}\eta_\alpha\eta_{\alpha'}\omega_{\substack{\alpha\\(-\alpha')}}\frac{\omega_c J_l(\xi)J_{l-m}(\xi)}{\Delta_{l(l-m)}+ia}, \tag{7}$$

where $J_l(\xi)$ is a Bessel function and $\xi=\mu\mathbf{q}\mathbf{E}_2/\Omega$. As seen from (4)–(7), the maximum response $u_{-\alpha'}$, should result not only from the smallness of the diffusion current ($\omega_\alpha\ll\omega_D$),[1] but also from the smallness of the Maxwellian screening of the field $E_2(\omega_\alpha\gg\omega_c)$, and should occur at definite values of the constant drift (5). The physical meaning of the selection rules of[1] follows directly from the drift-resonance conditions (5) and (6). Thus, without a drift ($\mathbf{E}_0=0$) the condition (5) is satisfied for $m\Omega=2\omega_\alpha$ when m is even:

$$m=2p, \quad \text{i.e., } p\Omega=\omega_\alpha(p=1,\cdots) \quad \text{at } v_0=0. \tag{8}$$

The number m can be arbitrary only if the drift velocity is near the value $v_\alpha/\cos\theta$[$l=0$ in (5)], and not simply at $\mathbf{E}_0\neq 0$[1]:

$$m\Omega=2\omega_\alpha \quad (m=1,\cdots) \quad \text{at } v_0=v_{0,m}(\theta)\equiv v_\alpha/\cos\theta. \tag{9}$$

This rule is valid also for "conversion" echo [$\alpha'\neq\alpha$ in (3)–(6)]. We note that from the maximum of the echo at $v_0\approx v_{0,m}$ we can estimate the carrier mobility: in the samples of[1] $u=v_\alpha/(\cos\theta E_0)=40$–$60$ cm²/V-sec.

It is typical that at small amplitudes E_2, when $\xi<1$, the ratio of the echo amplitudes at $m=2p$ [see (4) and (7)]

$$|u^p_{-\alpha}(\Delta_0=a)/u^p_{-\alpha}(v_0=0)|\approx p!e^{\Gamma_M\tau_2}/(\Gamma_M\tau_2(p+1)\cdot\cdots\cdot 2p) \tag{10}$$

does not depend on the field E_2 and is determined by the parameter $\Gamma_M\tau_2$, where $\Gamma_M\tau_2$, $\Gamma_M=\eta_\alpha^2\omega_\alpha/4(1+q^2r_0^2)$. It was this which was observed in experiment of[1], Fig. 1: for $\Omega=\omega_\alpha$ the ratio (10) is equal to 2.5 when E_2 is 12.9 and 6 dB lower than the amplitude of the voltage source. Owing to the large growth rate Γ_M, the echo appears at $v_0\cong v_\alpha/\cos\theta$ also for $m=3,4,5,6$ (Fig. 2[1]). The broad maxima in the dependence of the echo on the field E_0[1] are probably due to excitation, by the first pulse, of sound waves in the angle cone (Fig. 1) and therefore the maximum response $u_{-\alpha}(\theta_i)$ from the waves with different directions will occur at different values of $(v_0)_i=v_{l,m}(\theta_i)$ (Fig. 3). The quantity $v_{l,m}(\theta)$ in (5) can be smaller than the speed of sound $\tilde{v}_\alpha$, as for example the maximum of the echo $u_{-\alpha}(m=4)$ at $v_0\cong v_\alpha/2\cos\theta$ on Fig. 3.

At large amplitudes E_2, when $v_2/v_\alpha\sim 1, \mathbf{v}_2=\mu\mathbf{E}_2$, the coupling parameter $R^p_{-\alpha}\sim J_p^2(pv_2\cos\theta/v_\alpha)$ decreases slowly with decreasing $p(J_1^2(1)\approx 0{,}2,\ldots, J_8^2(10)\approx 0.1)$. Consequently, an echo can be observed [$u_{-\alpha}\propto R_{-\alpha}$ from (4)] at low subharmonics $\Omega=\omega_\alpha/p, p\gg 1$.

The maxima of the holographic echo at $\Omega=\omega_1/m$[1] can also be attributed to the presence of carrier plasma in the region of the drift resonance (5)–(6).

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1)Shiren and Melcher[1] have attempted to explain this qualitatively as being due to ionization of the impurities in the alternating electric field, but the quantitative estimates exceed the experimental values of the fields by an order of magnitude.[1]

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