Localized states of the order parameter near a dislocation

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(Submitted 4 October 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **26**, No. 9, 624–627 (5 November 1977)

At a temperature above the ordering temperature of the volume phase, filaments of a new phase are produced along a dislocation. The dimensions of these regions and the temperature at which they occur are determined. In the case of a superconducting transition, the critical field and the current that destroys these local states are determined.

PACS numbers: 61.70.Ga, 61.50.Ks, 74.40.+k

1. Dislocations, having a stress field, interact via this field with an ordered system (spin system, system of Cooper pairs). In the simplest case this interaction is described by terms of the form $u_{ii}(\mathbf{r})\eta^2(\mathbf{r})$, (where u_{ik} is the strain tensor and $\eta(\mathbf{r})$ is the order parameter (for an edge dislocation $u_{ii}=(b/2\pi r)(1-2\sigma)/(1-\sigma)$, where r is the distance to the dislocation axis, $\mathbf{b}$ is the Burgers vector, and σ is the Poisson coefficient[1]). These terms result from the dependence of the ordering temperature on the density of the material. Owing to the slow decrease of u_{ii} (and the presence of a alternating-sign factor in it), an extensive region exists near the dislocation, where the transition can occur at a temperature T_0 higher than the temperature T_c of the transition in the volume. We calculate below the values of T_0, of the free energy F of the resultant localized state, as well as the behavior of $\eta(\mathbf{r})$ near the dislocation. We consider primarily a superconducting phase transition and estimate the critical field and the current for the localized state.

2. We write down the free energy in the form of a Ginzburg–Landau (G–L) functional:

$$F=\int d^2\mathbf{r}dz\{C|\nabla\eta|^2+(-\alpha_0\tau+\alpha_0U_1(\mathbf{r}))|\eta|^2+(\beta/2)|\eta|^4\}, \quad (1)$$

$$\tau=(T-T_c)/T_c;\quad U_1(r)=-B\sin\phi/r;$$
$$B=\frac{b}{2\pi}\frac{1-2\sigma}{1-\sigma}\frac{\partial\ln T_c}{\partial\ln V}, \quad (2)$$

where T is the temperature, V is the specific volume, and C, $\alpha(T)$, and β are coefficients in the expansion of the free energy. Alternately, we use the more convenient dimensionless form:

$$F=F_0\int d^2\vec{\rho}\, d\zeta\{|\nabla_{\vec{\rho}}\psi|^2+(U_1(\vec{\rho})-E))|\psi|^2+\tfrac{1}{2}|\psi|^4\},$$
$$\rho=r/l;\quad \psi=-\eta\sqrt{\beta C}/\alpha_0 B;\quad \zeta=z/l, \quad (3)$$
$$U_1(\vec{\rho})=-\sin\phi/\rho;\quad E=\tau C/\alpha_0 B^2,$$
$$l=-C/\alpha_0 B;\quad F_0=|\alpha_0|BC/\beta;\quad \alpha_0\equiv\alpha(0).$$

The problem consists of minimizing the functional (3). It is required to find the minimal value of $E^{(0)}$ at which F is also negative. The functional (3) corresponds to a G–L equation

$$-\Delta\psi+[(U_1(\vec{\rho})-E)+|\psi|^2]\psi=0. \tag{4}$$

[o find $E^{(0)}$ it suffices to solve (4) without the nonlinear term. The dislocation potential $U_1(\vec{\rho})$ loes not admit of separation of the variables, owing to the presence of the factor $\sin\phi$. We have onsidered, besides U_1, two potentials that simulate the dislocation potential:

$$U_2(\rho)=-1/\pi\rho; \qquad U_3(\vec{\rho})=\begin{cases}2U_2(\rho) & 0<\phi<\pi \\ \infty & \pi<\phi<2\pi\end{cases}.$$

'he potentials are so "normalized" that their values averaged over the angles are the same in the egion of the well. The potential U_2 does not take into account the asymmetry in ϕ, while U_3 is symmetrical to the utmost. The true potential has intermediate asymmetry. It is obvious that

$$E_2^{(0)}<E_1^{(0)}<E_3^{(0)}.$$

'he potential U_2 corresponds to a linear equation whose eigensolutions are the functions $\psi^{cs}_{nm\kappa}(\rho/\pi)$ (non-normalized)[2]

$$\psi^{cs}_{nm\kappa}(\rho)=\exp(i\kappa\zeta)\begin{Bmatrix}\cos m\phi\\ \sin m\phi\end{Bmatrix}\rho^m_{nm}\exp(-\rho_{nm}/2)$$

$$\times\Phi(-n,2m+1;\rho_{nm});\ \rho_{nm}=\rho/(n+m+1/2);\ n,m=0,1,2,\cdots, \tag{5}$$

'here $\Phi(a,b,x)$ is a confluent hypergeometric function. The "energy levels" have a Coulomb" form:

$$E_{nm\kappa}=-[\pi(2n+2m+1)]^{-2}+\kappa^2. \tag{6}$$

1 particular, $E_2^{(0)}=E^c_{000}=-1/\pi^2; \psi_2^{(0)}=\exp(-\rho/\pi)$. The eigensolutions for the potential U_3 are of ıe form $\psi^s_{nm}(2\rho/\pi)$

$$E_3^{(0)}=E^s_{010}=-4/9\pi^2; \qquad \psi_3^{(0)}=\sin\phi\,\theta(\phi)\rho\exp(-\rho/3).$$

.n approximate solution with a potential U_1 was obtained by us using a variational procedure and ‹panding the trial function in powers of $\psi^{cs}_{nm\kappa}$. For four trial functions $E_1^{(0)}=-0.089$, i.e., (5) is ıdeed satisfied. We have $\psi_1^{(0)}\sim f(\rho,\phi)\exp(-\rho)$, where $f(\rho,\phi)\sim 1$ and has somewhat lower ıisotropy than $\psi_3^{(0)}(\rho,\phi)$.

It can be shown that at $\Delta E=(E-E_i^{(0)})/E_i^{(0)}\ll 1$ the solution of the nonlinear equation takes the ırm $A(E)\psi_i^{(0)}+O((\Delta E)^{3/2})$. Minimization of the functional (3) with respect to A yields:

$$A^2=\frac{\langle|\psi_i^{(0)}|^2\rangle}{\langle|\psi_i^{(0)}|^4\rangle}(E-E_i^{(0)}); \qquad F=-\tfrac{1}{2}F_0A^4\langle|\psi_i^{(0)}|^4\rangle\frac{L}{l};$$

$$E\geqslant E_i^{(0)}, \tag{7}$$

here L is the dislocation length and $\langle x\rangle=\int x d^2\vec{\rho}$. In particular, for the true dislocation potential e have $F\approx-83F_0(E-E_1^{(0)})^2L/l$ i.e., in this approximation we have a jump of the heat capacity at temperature:

$$T_0=T_c\left(1+\frac{\alpha_0E_1^{(0)}B^2}{C}\right). \tag{8}$$

n all the estimates that follow we assume that formulas (7) are valid also at $\Delta E\sim 1$].

3. For superconductors, the presence of localized states means the appearance of supercon- ıcting filaments at a temperature $\Delta T\approx 0.16T_c(B/\xi_0)^2$ higher than critical. The characteristic

dimension of these filaments is $l_0 = \sim(\xi_0^2/B) \gg \xi_0$, so that the G–L equations can be applied to these states.

The destruction of the superconductivity in filaments takes place at an upper critical field $H_{c_2}^{(1)}$.[3] We consider the case when the magnetic field is directed parallel to the dislocation axis. Writing down in the usual manner the G–L equation with a field, the dislocation potential can be taken into account in the limiting case of a "strong" field $\nu \gg 1$, ($\nu = el^2H/\hbar c$, c is the speed of light) by using perturbation theory. In the opposite limiting case, the magnetic field is taken into account by perturbation theory. For the true dislocation potential, the dependence of ν on E takes the form

$$\nu(E) = 0.13\sqrt{E - E_1^{(0)}} \qquad (E - E_1^{(0)})/E_1^{(0)} \ll 1$$

$$\nu(E) = 0.05 + 0.5E + 0.23\sqrt{E} \qquad E \gg 1. \tag{9}$$

Extrapolation of both relations to $E = 0$ yields $\nu \approx 0.045$, which corresponds to fields $H_{c_2}^{(1)} \approx 0.26\Phi_0(B/\xi_0^2)^2$, where Φ_0 is the flux quantum.

Current can flow along the superconducting regions that are produced near the dislocations. The critical value of this current for one dislocation has been estimates by us for the potential U from the conditions

$$H_c(r) = H_c|\eta(r)|^2 = 2I/rc; \qquad -\frac{\partial H_c(r)}{\partial r} = 2I_c/r^2c \tag{10}$$

so that $I_c \approx 0.09c(E - E_2^{(0)})BH_c$.

For Nb_3Sn, assuming $\xi_0 \approx 65$ Å, $B \approx 10$ Å, $H_c \approx H_{c_1} \approx 300$ G, we obtain:

$\Delta T \approx 0.07$ K; $l_0 \approx 230$ Å; $H_{c_2}^{(1)} \approx 3000$ G; $I_c \approx 10^{-6}$ A.

The local states can be revealed experimentally, for example, by the abrupt decrease of the resistance at the points where the dislocations emerge to the surface.

The authors thank Academician I.M. Lifshitz for interest in the work and for a useful discussion.

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