

# Backward elastic waves in plates

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Elastic waves with oppositely directed phase and group velocities were excited and investigated in experiment.

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Among the roots of the dispersion equations for elastic waves in bounded solids, there are some with corresponding waves that are characterized by opposite directions of the phase and group velocities  $V_{ph}$  and  $V_g$ . These are the so-called backward waves (ordinary waves with  $V_{ph}$  and  $V_g$  in the same direction will henceforth be called direct waves). Backward waves can exist, for example, in plates at certain values of the ultrasound frequency and plate thickness.<sup>[1,2]</sup> These waves have not been studied experimentally. The present paper is devoted to an experimental observation and investigation of backward waves in piezoelectric plates.

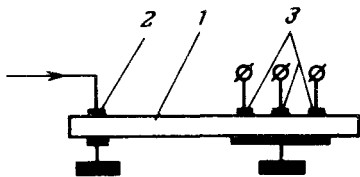


FIG. 1. Experimental setup.

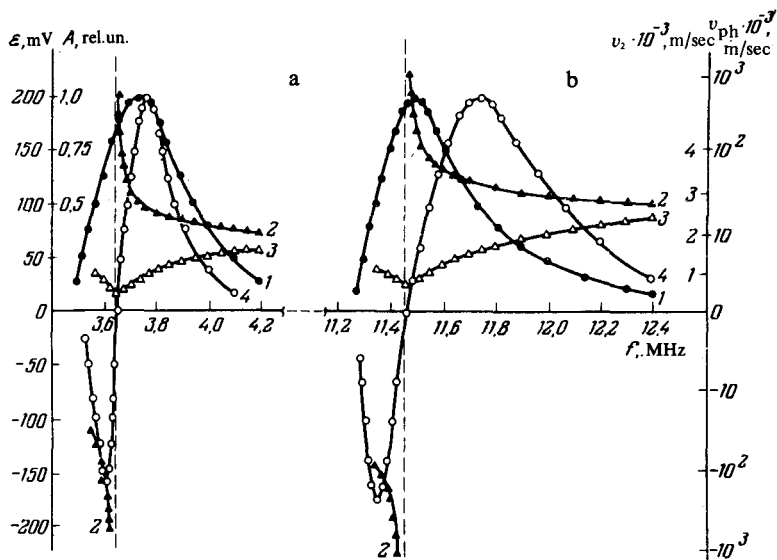


FIG. 2. Frequency dependences, near the critical frequencies, of the amplitudes ( $A$ ) of the excited elastic waves (curve 1), of the phase velocity ( $v_{ph}$ , curve 2), of the group velocity ( $v_g$ , curve 3), and of the acousto-emf ( $\mathcal{E}$ , curve 4). a) Mode  $s_2$  in TsTS plate; b) first resonant mode in  $\text{LiNbO}_3$  plate with  $h=195 \mu\text{m}$ . The ordinates of curve 4(a) are magnified tenfold.

An analysis of the dispersion equations for elastic waves in isotropic plates shows that only resonant modes of the Lamb type can be of the backward type, and furthermore at frequencies below critical  $f_{cr}$ , where they are described in the main by complex wave numbers ( $k$ ). Modes with real " $k$ " (direct waves) go over continuously into modes with complex " $k$ " (backward waves).<sup>[3]</sup> At the chosen propagation direction of the elastic energy (group velocity), the phase velocity can reverse sign on going through the critical frequency. Accordingly, to observe backward waves in the experiment we studied resonant modes of Lamb waves near the critical frequencies. The experiments were performed on  $yz$ -cut  $\text{LiNbO}_3$  plates,  $h=195 \mu\text{m}$  or  $h=84 \mu\text{m}$  thick, and on polarized ceramic TsTS plates with  $h=645 \mu\text{m}$  and the polarization axis perpendicular to the plate surface. We studied the frequency dependences of the elastic-wave excitation (the transducer output) of the elastic waves, of the phase and group velocity, and of the longitudinal acousto-emf in silicon placed on the surface of the piezoelectric plate (by knowing the sign of the acousto-emf and the type of carriers we can uniquely determine the direction of the phase velocity<sup>[4]</sup>). The experimental setup is shown in Fig. 1. The electrodes 2 were used to excite the elastic waves. To measure the phase velocity we used electrodes 3 with a distance between them shorter than the smallest wavelength encountered in the measurements. The phase velocity was measured by comparing the phases of the oscillations of the voltage picked off the electrodes 3. The accuracy in the determination of the phase velocity was 1%. The group velocity was determined by two methods: by measuring the delay time of the ratio pulses in the plate (with accuracy 15%), and by determining the dependence of the phase shift ( $\phi$ ) on the frequency ( $f$ ) in a plate of length  $l$ , since  $V_g = d\omega/dk = 2\pi l(df/d\phi)$ . The accuracy with which the group velocity was measured by the second method was much higher, 1–2%. To measure the longitudinal acousto-emf we placed on the surface of plate 1, instead of the electrodes 3, a  $p\text{Si}$  plate with resistivity  $\sim 7500 \Omega\text{-cm}$  and thickness 1.4 mm.

In the  $\text{LiNbO}_3$  samples, only the first resonant ( $r_1$ ) Lamb-wave mode was adequately excited (in  $yz$ -cut  $\text{LiNbO}_3$  plates the Lamb waves do not break up into symmetrical and antisymmetrical ones). The loss to double conversion ( $P$ ) at the maximum of the excitation of this mode was  $P=12$  dB. In the TsTS plate, the easiest to excite was the second symmetrical ( $s_2$ ) Lamb-wave mode ( $P=20$  dB), as against  $P=32$  dB for the first antisymmetrical ( $a_1$ ) mode. Investigations of these modes have shown that the  $a_1$  mode is excited in TsTS only at frequencies above critical and that  $V_{ph}$  increases monotonically in the entire excitation region, while  $V_g$  decreases as  $f_{cr}$  is approached, both velocities having the same direction. On the other hand, the  $r_1$  mode in  $\text{LiNbO}_3$  and the  $s_2$  mode in TsTS were also excited at frequencies below critical (the critical frequencies were determined from the resonance frequencies in the plate). At  $f > f_{cr}$  the  $r_1$  and  $s_2$  modes behave in analogy with the  $a_1$  mode, and at  $f < f_{cr}$  their properties differ appreciably). On going through  $f_{cr}$   $V_{ph}$  reverses direction jumpwise and decreases with further decrease of  $f$ , and the acousto-emf also reverses sign. At frequencies  $f < f_{cr}$  the sign of the acousto-emf corresponded to oppositely directed  $V_{ph}$  and  $V_g$ .  $V_g$  has a minimum at  $f=f_{cr}$ . The foregoing is illustrated in Fig. 2, which shows the experimental results near  $f_{cr}$ , the frequency dependences of the excitation (1), of the phase velocity (2), of the group velocity (3), and of the acousto-emf (4) for the modes  $s_2$  and  $r_1$  in a TsTS plate (a) and in  $\text{LiNbO}_3$  with  $h=195 \mu\text{m}$  (b). The positions of the critical frequencies are marked by vertical dashed lines. It is seen that on going through the critical frequency the phase velocity of the modes  $s_2$  and  $r_1$  reverses sign, and consequently the acousto-emf reverses sign.

Thus, the entire aggregate of the experimental data indicates that elastic waves with oppositely directed phase and group velocities can be excited in the plates.

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