Role of conduction electrons in the formation of the thermal resistance of a metal–dielectric interface

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The size effect in the heat transfer from electrically heated metallic plates (films) is discussed for the case when the electronic contribution to the thermal resistance of the metal–dielectric interface is decisive.

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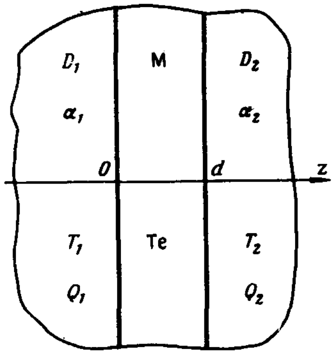
1. In the interpretation of experiments on the thermal resistance of a metal–dielectric (M–D) interface it is customary at present to use the acoustic-mismatch theory.[1] The experimentally measured temperature discontinuity as a function of the heat flux through the M–D interface is connected in this approach only with the acoustic characteristics of the metal (density and speed of sound s).[2] It is shown in the present paper that this interpretation is valid only for sufficiently bulky metallic samples. In the opposite limiting case of thin plates (films)—the appropriate estimates are given below—the role of the electrons in the formation of the temperature discontinuity on the M–D interface becomes decisive. To explain the physical cause of this increase of the electronic contribution with decreasing metal thickness (configuration of the sandwich type D_1–M–D_2, see Fig. 1) we consider qualitatively the heat-transfer mechanism through the M–D interface.

Although the electrons are the principal carriers of the heat in the metal, they cannot penetrate into the dielectric, so that heat transfer through the M–D interface is effected by the phonons and depends on the acoustic transparency of the interface. On the metal side there exists therefore near the interface a transition layer in which the thermal energy transported by the electrons is transformed into a phonon flux. The thickness of this layer, as can be easily seen, is of the order of the mean free path of the thermal phonon relative to scattering by electrons l_{pe} ($l_{pe} \sim \hbar v_F/kT \sim 10^{-3}$–$10^{-4}$ cm at helium temperatures). It is intuitively obvious that if $d \gg l_{pe}$ (d is the thickness of the metal plate), then the detailed structure of this transition layer is of no importance



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n the calculation of the thermal resistance of the M–D interface; this conforms in fact to the usual ıpproach, in which the electronic transition is neglected. On the other hand, if $d \lesssim l_{pe}$, then a kinetic calculation of the transition layer is essential. A brief description of such a calculation, with ıccount taken of the finite transparency of the M–D interface to the ponons, is given in Sec. 2. An analysis of the results of the calculation and a discussion of the experimental situation are given in Sec. 3.

2. Consider a metallic plate (film) of thickness d, in contact on both sides with bulky dielectrics D_1 and D_2 whose temperatures T_1 and T_2 are known (see Fig. 1). Assume that direct current of density j flows through the plate, so that a power j^2/σ is released in a unit volume, where σ is the conductivity of the metal. It is required to find the heat fluxes Q_1 and Q_2, given the transparencies α_1 and α_2 of the interfaces to phonons.

We choose the z axis to be perpendicular to the interfaces of the media and assume that the problem is spatially homogeneous in the oxy plane. We assume also that the distribution of the electrons in the plate is characterized by a temperature T_e, as yet unknown. If $d < l_{pe}$, then it is easy to show that the inhomogeneity of T_e along the z axis can be neglected to the extent that the parameter $\kappa_p/\kappa_e \gg 1$ is small, where κ_p and κ_e are the heat conduction coefficients of the phonons and electrons, respectively. At the same time, the phonon distribution function $N(\mathbf{q},z)$ (where $\mathbf{q}$ is the wave vector of the phonon) can be essentially inhomogeneous, and must be determined from the kinetic equation

$$s_z \frac{\partial N}{\partial z} = \hat{\nu} W \tag{1}$$

with appropriate boundary conditions. Here s_z is the projection of the phonon velocity on the z axis, and $\hat{\nu} W$ is the phonon-electron collision integral, which in our case reduces to the expression $\nu_{pe}[n(T_e)-N(z)]$, where $n(T_e)$ is the equilibrium Bose function with temperature T_e, while ν_{pe} is the average frequency of the collisions between the phonons of frequency ω and the electrons. It is important in what follows that ν_{pe} is determined by the electron-phonon interaction and for the Debye model it is proportional to ω ($\nu_{pe} \sim \omega s/v_F$ where v_F is the Fermi velocity). Omitting the intermediate calculations and the analysis for the expression in the case $N \lessgtr (\mathbf{q},z)$, we present the final result for the heat flux Q_1 (the expression for Q_2 differs from it only in that the indices 1 and 2 are interchanged and the sign is reversed)

$$Q_1 = \sum_{q_z > 0} \alpha_1 \gamma \{(1-\beta_2 x^2)[n(T_1) - n(T_e)] - \alpha_2 x[n(T_2) - n(T_e)]\}, \tag{2}$$

where $x = \exp(-d/l)$, $l \equiv s_z/\nu_{pe}$, $\gamma \equiv \hbar\omega_q s_z/(1-\beta_1\beta_2 x^2)$, and $\beta \equiv 1-\alpha$.

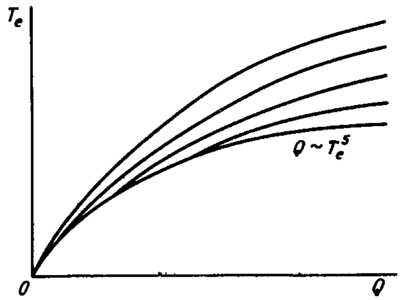

FIG. 2.

The value of T_e is determined from the balance equation for the electrons $Q=Q_1-Q_2$, wher $Q\equiv j^2d/\sigma$ is the total density of the heat flux passing through the interfaces. Relation (2), i: particular, makes it possible to analyze the thermal resistance of a D_1–M–D_2 sandwich withou internal heat sources ($T_1\neq T_2$, $Q=0$), a problem that may be of independent interest. Relegatin this analysis to a more detailed communication, we turn to a question of importance in experi ments with heat pulses,[1], namely the radiation from an electrically heated metal plate (film) int a medium with temperature T. The sought connection between Q and T_e, which follows from (2) can be represented at $T_1=T_2\equiv T$ in a compact form by introducing the effective combine transparency of the interfaces

$$Q=\sum_{q_z>0}\hbar\omega_q s_z\tilde{\alpha}(\mathbf{q},d)[n(T_e)-n(T)], \quad (3$$

$$\tilde{\alpha}\equiv(1-x)[\alpha_1(1+\beta_2x)+\alpha_2(1+\beta_1x)]/(1-\beta_1\beta_2x^2). \quad (4$$

We note that in contrast to the "bare" transparencies α_1 and α_2, which to simplify the formula were assumed to be independent of the phonon incidence angle on the interface, the effectiv transparency $\tilde{\alpha}$ depends on $\mathbf{q}$ (via the quantity x).

3. Let us analyze expressions (3) and (4) in greater detail, since they contain the connectio between the experimentally observed $T_e(Q)$ dependence (see articles 12 and 13 in[1], as well as th more detailed exposition in[2]) with the film thickness and the transparency parameters. W determine first the temperature-dependent parameter $\epsilon\equiv 2d\beta_1\beta_2/l_{pe}(1-\beta_1\beta_2)$, so that $\epsilon\sim T_e$. Th formulas corresponding to the usual interpretation,[1,2] in which the electron contribution to th thermal resistance of the interface is neglected, are obtained from (3) and (4) at $\epsilon>2$. Indeed, i this limit we have $\tilde{\alpha}=\alpha_1+\alpha_2$ and at $T_e\ll\theta_D$ we obtain from (3) the well known resul $Q=A(T_e^4-T^4)$, where the constant A is determined only by the acoustic characteristics of th metal. The thermal radiation of the film is at equilibrium (with a temperature T_e) and th maximum of the spectral intensity corresponds to the frequency $\omega_m\approx 2.8T_e$. If Q is constant, the T_e does not depend on d.

We consider now the inverse limiting case $d\ll l_{pe}$, but still assume that $d>\lambda$, where λ is th wavelength of the thermal phonon, so as to neglect the deformation of the phonon spectrum of th film (we note that $\lambda/l_{pe}\sim s/v_F\ll 1$). In that case, if $\epsilon\ll 1$, then T_e does not depend on the trans- parency at all. Indeed, most phonons radiated by the electrons manage then to leave the film (anc are not "readsorbed" inside the film), and the electrons and the lattice can be described with th aid of two different temperatures T_e and T.[3,4] Just as in[3] we have $Q=B(T_e^5-T^5)$, where th constant B is connected with the electronic charcteristics of the metal. The phonon emission from the film is no longer at equilibrium, and the maximum of its spectral intensity corresponds to $\omega_m\approx 3.9T_e$. At constant Q we have $T_e\sim d^{-1/4}$, i.e., it depends on the film thickness, albeit weakly.

A simple analysis of (3) shows that in the general case $T_e(Q)$ is monotonic and "convex pward." The approximate form of these relations at different values of the transparency of one of ıe boundaries is shown in Fig. 2. The curves with the smaller slope correspond to lower transarency. The transition from the T^4 to the T^5 law occurs at $\epsilon \sim 1$. An important feature of the urves in Fig. 2 is that the $Q \sim T_e^5$ curve, which corresponds to the case of pure electronic ıperheating, lies lower than the remaining curves (with the exception of the sections where they oincide). This means that in the limit $\epsilon \ll 1$ the cooling of the film is most effective (the minimal alue of T_e at the given Q). It should be noted that in experiment, in a number of cases,[2,5] the ıatching of the film to the substrate is better than would follow from the theory of the acoustic ıismatch (which in our case corresponds to the sections of the curves with $\epsilon \gtrsim 2$). To interpret ıese results, the authors of[2,5] use the so called absolute black body (ABB) model, for which there ; no theoretical physical basis whatever in this case. As follows from the foregoing, it is actually ossible to have a regime in which the heat removal is maximal and independent of the transparncy coefficients, as long as $\epsilon \ll 1$. However, in contrast to the ABB model, in the case of electronic ıperheating the cooling is determined primarily by the intensity of the electron-phonon ıteraction.

We make one more remark that can explain the "anomalous" transparency results obtained ɔr films sputtered at room temperature.[2] The customarily[2] investigated film is sputtered on a ooled substrate and therefore has a certain structural disorder (for In, Sn, and Al films this can asily be seen from the small increase of the temperature of the superconducting transition). This amorphization" of the film can make it very difficult for the phonons to leave the film, and onsequently make it difficult to satisfy the condition $\epsilon \ll 1$. On the other hand, if the films are close ı structure to the bulky metal (they were sputtered at room temperature or were well annealed) nd the bare transparencies α are not very small, then, as shown by estimates, satisfaction of the ondition $\epsilon \ll 1$ can be easily reached, and this might explain the results of[2] without resorting to the ιBB model.

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