Mechanism of "hard" excitation of spin waves in antiferromagnets

V. S. Lutovinov and M. S. Savchenko

Institute of Radio Engineering, Electronics, and Automation
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It is shown that relativistic exchange-enhanced magnon-phonon interaction can lead to hard excitation of spin waves in antiferromagnets. The obtained jump $\Delta h = h_{c1}-h_{c2}$ of the threshold field agrees, in order of magnitude and in its dependence on the temperature and on the magnetic field, with the experimental data [1–3].

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When spin waves (SW) are parametrically excited in antiferromagnets (AF), a "hardness" effect is observed—the SW parametric excitation begins and ends at different values of the microwave fields h_{c_1} and h_{c_2} ($h_{c_1} > h_{c_2}$) respectively.[1–3]

The purpose of the present study was to identify the physical mechanism that causes the hard character of the excitation of the SW. It is shown that the relativistic interaction of the magnon and the phonon subsystems, enhanced by exchange interaction,[4,5] can explain the considered phenomenon.

It is known that the threshold microwave-field intensity h_c is determined by the usual relation $h_c = \gamma_m / U$ [$U = \mu^2(2H + H_D)/4\epsilon_{\mathbf{k}}$ is the coupling coefficient of the magnons with the microwave field], where γ_m is the damping decrement of the excited magnons, which can receive contributions, in particular, from the magnon-phonon interactions. This contribution γ_{m-ph} is determined by the expression

$$\gamma_{m-ph}(\mathbf{k}) = 8\pi v_0 \int \frac{d\mathbf{q}}{(2\pi)^3} \left| \psi_{m-ph}(\mathbf{k},\mathbf{q}) \right|^2 (n_m(\epsilon_{\mathbf{q}-\mathbf{k}}) - n_{ph}(\omega_{\mathbf{q}}))\delta(\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{q}-\mathbf{k}} - \omega_{\mathbf{q}}), \quad (1)$$

where

$$\psi_{m-ph}(\mathbf{k},\mathbf{q}) = \theta \sqrt{\frac{\omega_{\mathbf{q}}}{2Mc^2}} \frac{J_0}{\sqrt{\epsilon_{\mathbf{k}}\epsilon_{\mathbf{q}-\mathbf{k}}}}, \qquad \epsilon_{\mathbf{k}} = \sqrt{\epsilon_0^2 + s^2 k^2}$$

is the amplitude of the process of coalescence of two magons into a phonon,[1)] [6] $\theta = Bv_0 \sim 5$ K is the characteristic magnetoelastic energy, B is the magnetoelastic constant, v_0 is the volume, M is the mass of the unit cell, c is the speed of sound, and J_0 is the exchange constant.

An analysis of the conservation laws for low-temperature AF shows that the process of the coalescence of two magnons into a phonon is allowed if the energies $\omega_{\mathbf{q}}$ of the intermediate and $\epsilon_{\mathbf{q}-\mathbf{k}}$ of the magnons satisfy the relations

$$\omega_- \lesssim \omega_{\mathbf{q}} < \omega_+, \qquad \epsilon_- \lesssim \epsilon_{\mathbf{q}-\mathbf{k}} \lesssim \epsilon_+, \quad (2)$$

where

$$\omega_\pm \approx 2\epsilon_{\mathbf{k}} \pm 2ask, \qquad \epsilon_\pm \approx \epsilon_{\mathbf{k}} \pm 2ask, \qquad a = s/c.$$

Taking the relations (2) into account, we can represent (1) in the form

$$\gamma_{m-ph}(k) \approx \eta(aN_m - N_{ph}), \tag{3}$$

here

$$\eta \approx 2\pi\frac{c}{s}\frac{\theta^2 J_0^2}{Mc^2 s k\epsilon_{\mathbf{k}}} \qquad a = \frac{k_{ph}^2}{k_m^2}\frac{\Delta k_{ph}}{\Delta k_m} \tag{4}$$

$N_m=(v_0 k_m^2\Delta k_m/2\pi^2)\bar{n}_m$ and $N_{ph}=(v_0 k_{ph}^2\Delta k_{ph}/2\pi^2)\bar{n}_{ph}$ are the number of the magnons and phonons in the energy interval (2) per unit cell, while Δk_m and Δk_{ph} are the widths of the **k**-space intervals for the magnons and phonons respectively.

In the case when the system is parametrically excited, the occupation numbers of the magnons and phonons in the energy intervals (2) should be determined from the kinetic equations. For N_m and N_{ph}, taking into account the coalescence of parametric magnons with the thermal ones with formation of a phonon, we have

$$\frac{dN_m}{dt} = -\gamma_m(\bar{N}_m - \bar{N}_m^0) - \Gamma\bar{N},$$
$$\frac{dN_{ph}}{dt} = -\gamma_{ph}(\bar{N}_{ph} - \bar{N}_{ph}^0) + \Gamma\bar{N}. \tag{5}$$

Here N_m^0 and N_{ph}^0 are the equilibrium values of N_m and N_{ph}, N is the number of parametrically excited magnons per unit cell, and Γ is the damping coefficient of the parametric magnon in the magnon-phonon relaxation process. It can be shown that in AF at $T<\theta_D(\theta/\theta_D)^{1/2}$ (θ_D is the Debye temperature) the value of γ_{ph} is determined not by the phonon-phonon but by the phonon-magnon processes, and in this case

$$\gamma_{m-ph}^0(\epsilon_{\mathbf{k}})/\gamma_{ph-m}^0(2\epsilon_{\mathbf{k}}) \approx 8a^3\frac{\epsilon_{\mathbf{k}}}{sk}f(T), \quad f(T) = e^{\epsilon_{\mathbf{k}}/T}\left(\frac{e^{\epsilon_{\mathbf{k}}/T}-1}{e^{2\epsilon_{\mathbf{k}}/T}-1}\right)^2.$$

Substituting the stationary solution of Eqs. (5) in relation (3), we obtain

$$\Gamma(N) = \frac{\gamma_{m-ph}^0}{1+\eta(1+\xi)N/\gamma_{ph-m}^0}, \qquad \xi = 4a^3\frac{\epsilon_{\mathbf{k}}}{sk}\frac{\gamma_{ph-m}^0}{\gamma_m}. \tag{6}$$

Thus, the damping of parametric magnons contains a component that depends substantially on N.[2)]

The stationary number N of parametric SW can be obtained from the condition[7]:

$$h^2U^2 = S^2N^2 + \gamma_m^2, \qquad S = J_0\frac{\epsilon_0^2 + 3(\mu H)^2}{4\epsilon_{\mathbf{k}}^2}. \tag{7}$$

Recognizing that $\gamma_m=\gamma'_m+\Gamma(N)$ (γ'_m is the contribution made to the magnon relaxation by all the processes with the exception of the magnon-phonon process), we obtain

$$h^2U^2 = S^2N^2 + \left[\gamma'_m + \frac{\gamma^o_{m-ph}}{1+\eta(1+\xi)N/\gamma^o_{ph-m}}\right]^2 . \qquad (8$$

From (8) we see that N is a non-single-valued function of h in a certain interval of microwave fields. The condition under which both roots coincide is of the form $d(h^2U^2)/d(S^2N^2)$. Hence assuming the parameter $\eta N/\gamma^0_{ph-m}$ to be small, we have at the single-value point

$$N_o = \frac{\gamma_m}{S}\frac{\eta}{S}\frac{\gamma^o_{m-ph}}{\gamma^o_{ph-m}} . \qquad (9$$

Substituting the value of N_0 in (8) we obtain

$$U^2h^2_{c_2} = \gamma^2_m\left[1-\left(\frac{\eta}{S}\right)^2\left(\frac{\gamma^o_{m-ph}}{\gamma^o_{ph-m}}\right)^2(1+\xi)^2\right], \qquad (10$$

where

$$\gamma_m = \gamma'_m + \gamma^o_{m-ph} = Uh_{c_1} . \qquad (11$$

The last relations for the jump $\Delta h = h_{c_1} - h_{c_2}$ at $\Delta h \ll h_c$ yields

$$\Delta hU \approx \gamma_m\left(\frac{\eta}{S}\cdot\frac{\gamma^o_{m-ph}}{\gamma^o_{ph-m}}\right)^2 . \qquad (12$$

An analysis of (12) shows that Δh in the region $T \lesssim 2$ K is a decreasing function of the temperature and of the magnetic field, in agreement with the experimental data of[1-3]. At characteristic parameter values $\theta \sim 5$ K, $sk \sim \epsilon_k \sim 0.5$ K, $J_0 \sim 30$ K, $Mc^2 \sim 10^5$ K, $H = 1$–5 kOe and $T \approx 1.5$ K we obtain from (12) a jump of order $\Delta h \approx 10^{-1}$–10^{-2} MHz, which also agrees with the experimental data.

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[1)]We omit the inessential dependence of the amplitude on the direction of the phonon wave vector and its polarization vector.

[2)]In principle, nonlinear damping of the type (6) yields also a three-magnon coalescence process (V_{3m}), with $\eta_{3m} \gtrsim \eta_{m-ph}$. However, parametric SW can interact in the V_{3m} process only with high-energy magnons ($k \sim k_{\rm Br}$). For this reason, $\eta_{3m}/\gamma_m(k_{\rm Br}) \ll \eta_{m-ph}/\gamma_{ph}$ and the V_{3m} processes make a negligible contribution to the jump Δh.

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