He^3–He II solutions in strong magnetic fields

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We investigate the properties of a low-density Fermi liquid, such as He^3–He II solutions, in strong magnetic fields, when the spin system is almost fully polarized. The most pronounced effects are connected with the tremendous growth of the kinetic coefficients.

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Degenerate solutions of He^3 in superfluid He^4 are the most typical examples of a Fermi liquid ow density. The properties of such systems are described by an expansion in $x^{1/3}$ (x is the ion concentration) and are specified, in the absence of the magnetic field, by only one ımeter—the s-scattering length a of the bare quasiparticles.[1–5]

In a strong magnetic field $\beta H \gg T_F > T$ (β is the magnetic moment of the He^3 atom, H is the ;netic field, T_F is the degeneracy temperature, and T is the temperature), the spins of practical-ll the He^3 atoms have the same direction. Owing to the identity of the fermions, the interaction ıld already be determined by p-scattering, in as much as s-scattering makes a contribution only he case of collisions of particles with oppositely directed spins.

In such a polarized solution, the interaction of the quasiparticles takes place on a Fermi ace of radius $p_F=(6\pi^2 N_3)^{1/3}\hbar$, with $p_F|a|/\hbar \ll 1$ (N_3 is the number of He^3 atoms per unit volume, le $|a|$ is of the order of the gas-kinetic dimension of the atom, 1.5 Å[5]). The energy spectrum of bare quasiparticle He^3 in superfluid He^4 is given by

$$\epsilon = -\Delta + \frac{p^2}{2M}\left[1 - \left(\frac{p}{p_c}\right)^2 \xi\right] - \beta H. \qquad (1)$$

e $\Delta \approx 2.8$ K is the energy gap, $M=2.3m_3$, m_3 is the mass of the He^3 atom, $p_c=m_4^s$, m_4 is the s of the He^4 atom, s is the speed of sound in He^4, and ξ is a rather small quantity.[6,7] The ›litude of p-scattering of two slow quasiparticles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ ($p_1=p_2=p_F$) in the .s. is determined by the angle of rotation ϕ of the relative momentum $\mathbf{p}=(\mathbf{p}_1-\mathbf{p}_2)/2$ [8]

$$f(\mathbf{p}_1', \mathbf{p}_2'; \mathbf{p}_1, \mathbf{p}_2) = B\frac{p^2}{M}\cos\phi, \qquad (2)$$

:re $\mathbf{p}'_1$ and $\mathbf{p}'_2$ are the momenta of the scattered particles ($p'_1=p'_2=p_F$), and $B \sim |a|^3$ is a constant. : Fermi-liquid function $f(\theta)$ (θ is the angle between the vectors $\mathbf{p}_1$ and $\mathbf{p}_2$) is determined by the vard-scattering amplitude (2) ($\phi=0$) [9]

$$f(\theta) = \frac{B}{M} p_F^2 \sin^2\frac{\theta}{2}. \qquad (3)$$

With the aid of the energy spectrum (1) and the Landau f-function (3) we can determine all thermodynamic properties of the He^3-He II solution in strong magnetic fields. Thus, the total ctive mass m^* of the excitations is equal to

$$\frac{m^*}{M} = 1 + 2\xi\left(\frac{p_F}{p_c}\right)^2 - \frac{1}{2} BN_3,$$

and the chemical potential μ_3 and the total energy of the solutions are given by

$$\mu_3 = -\Delta + \frac{p_F^2}{2M}\left[1 - \xi\left(\frac{p_F}{p_c}\right)^2 + \frac{4}{5} BN_3\right] - \beta H,$$

$$E = E_4^{(0)} - N_3\Delta + \frac{3}{10}\frac{p_F^2}{M} N_3\left[1 - \frac{5}{7}\xi\left(\frac{p_F}{p_c}\right)^2 + \frac{1}{2} BN_3\right] - \beta HN_3, \qquad (4)$$

where $E_4^{(0)}$ is the energy of pure He^4.

The propagation velocities of the hydrodynamic oscillations in a solution of He^3 in superfluid He^4 are determined formally by the same equation (5) as in the absence of a field, with account taken of relations (3) and (4). The speed of second sound increases in this case by a factor $2^{1/3}$, this being due not to the change of the interaction but to the increase of the radius of the Fermi sphere. There are no solutions of the zero-sound type, just as in the absence of a field, because the interaction is small.

The results of the transition from s- to p-scattering when the solution is polarized is the abrupt decrease of the interaction cross section. This leads to a considerable growth of the kinetic coefficients, whose value is inversely proportional to the interaction. The viscosity and thermal conductivity coefficients of a Fermi liquid are equal to[10,11]

$$\eta = \frac{64}{45} T^{-2}\frac{\hbar^3 p_F^5}{m^{*4}}\left\langle \frac{w(\theta,\phi)}{\cos(\theta/2)}(1-\cos\theta)^2\sin^2\phi\right\rangle^{-1} C(\lambda_\eta),$$

$$\kappa = \frac{8\pi^2}{3} T^{-1}\frac{(\hbar p_F)^3}{m^{*4}}\left\langle \frac{w(\theta,\phi)}{\cos(\theta/2)}(1-\cos\theta)\right\rangle^{-1} H(\lambda_\kappa),$$

where $w(\theta,\phi)$ is the particle-scattering probability and is connected with the scattering amplitude (2) by

$$w(\theta,\phi) = \frac{\pi}{2\hbar}\left(\frac{B}{M}\right)^2 p_F^4 \sin^4\frac{\theta}{2}\cos^2\phi, \qquad (5)$$

$\langle\cdots\rangle$—denotes averaging over the angles, while the coefficients $C(\lambda_\eta)$ and $H(\lambda_\kappa)$ [11] for the function w [Eq. (5)] are equal to $C(\lambda_\eta)=0.79$, $H(\lambda_\kappa)=0.55$. We ultimately get

$$\eta T^2 = \frac{7}{\pi}\left(\frac{\hbar^2}{MB}\right)^2 p_F\, 0.79; \qquad \kappa T = \frac{35\pi}{6}\left(\frac{\hbar^2}{MB}\right)^2\frac{1}{p_F}\cdot 0.55,$$

i.e., they have entirely different concentration dependences and differ from the values without the field[5] by the large factor $(x)^{-4/3} \gg 1$. So strong an increase of the relaxation time and of the mean free path of the excitations leads to a noticable narrowing of the NMR line and to a deceleration of the impurity atoms by the wall even in broad capillaries.

The condition $\beta H \sim T_F$ for total polarization of the solution corresponds to the relation

H[kOe]$\approx 5.2\times 10^4\ x^{2/3}$. Fields of the order of 100 kOe polarize solution with concentrations $x\leqslant 10^{-4}$. The temperature is then $T\ll T_F\leqslant 8$ mK, and the kinetic coefficients increase in comparison with their values in the absence of the field[5] by approximately 10^5 times. The viscosity, for example, reaches a value on the order of 10^{-2} poise, which is approximately equal to the viscosity of water, i.e., the superfluid liquid becomes simultaneously anomalously viscous.

The results for solutions in arbitrary magnetic fields, when the spin system is only partially polarized, and also of certain other types of Fermi systems, will be published later.

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