Effect of external plasma on the decay of a neutral current sheet

S. V. Bulakov, N. V. Sasorov, and S. I. Syrovatskiĭ

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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Decay of a current sheet (CS) leads to production of reclosing magnetic-field regions that travel along the sheet with super-Alfven velocity. Their dimension is determined by the electric conductivity in the plasma outside the CS.

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1. Current sheets (CS) attract attention as a possible site of effective dissipation of the magnetic field in a high-conductivity plasma.[1-3] It is shown in[4] that tearing instability[5,6] leads in some cases to the decay of the sheet and to the appearance of electric fields that accelerate charged particles.

From experiments on the study of the CS[7] it follows that the tear in the sheet has a threshold. The reason is the stabilizing effect of the external plasma.[8] We investigate below the influence of the plasma on the dynamics of the decaying CS.[1)]

Consider a case when the field does not penetrate into the CS (the characteristic scales exceed the sheet thickness L and the EM field screening dimension). For our problem, it is immaterial what assumptions are made concerning the sheath thickness ($L > r_H$ or $L < r_H$) and concerning the nature of the tearing instability (resistive[5] or collisionless mode[6]), although the condition under which the tear occurs depends on the assumption made.

2. The magnetic field of an infinite CS with a tear of width $2a$ (Fig. 1), as a function of the complex variable $\zeta = x + iy$, is given by[9]

$$\mathbf{H} = H_x + iH_y = H_0 \frac{\zeta}{(a^2 - \zeta^2)^{1/2}} . \quad (1)$$

Integrating the tensor of the Maxwellian stresses over the surface of the CS, we obtain the force per unit length at the point $\zeta = \pm a$ [9]

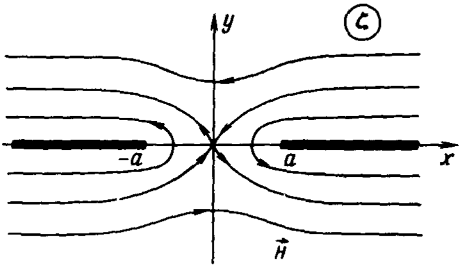

FIG. 1.

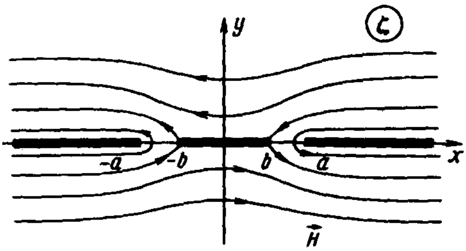

FIG. 2.

$$F(a) = H_0^2 a/2. \tag{2}$$

Neglecting thermal effects, allowance for which does not change the conclusions concerning the character of the motion[4] we write down the equation of motion of the end of the CS

$$\frac{d}{dt} M(a) \frac{da}{dt} = F(a) \tag{3}$$

$M(a)$ is the mass of the entrained plasma, which can be estimated at $M = n_s m_i a L$. The subscripts s and p pertain to the sheet and to the external plasma, respectively. We find thus that the CS is uniformly accelerated

$$\dot{a}(t) = \pi v_A^2 t/L; \qquad v_A = H_0/(4\pi n_s m_i)^{1/2}. \tag{4}$$

3. Allowance for the external plasma $[M(a) = n_s m_i a L + n_p m_i a^2]$ restricts the velocity $\dot{a}(t)$ to the Alfven velocity outside the layer $v_A^* = H_0/(4\pi n_p m_i)^{1/2}$, and leads to the appearance of a screening current in the tear at the location of the null line. The magnetic configuration is transformed into that shown Fig. 2 and is described by the expression

$$\mathbf{H} = H_x + iH_y = H_0\left(\frac{\zeta^2 - b^2}{a^2 - \zeta^2}\right)^{1/2} \tag{5}$$

The tension of the force lines at the point $\zeta = \pm b$ is equal to zero, while at the point $\zeta = \pm a$ we have

$$F = H_0^2 (a^2 - b^2)/2a. \tag{6}$$

The condition for the conservation of the magnetic flux at the tear[2)] is

$$\delta\Phi = \mathrm{Re}\left[iH_0 \int_b^a \left(\frac{\zeta^2 - b^2}{a^2 - \zeta^2}\right)^{1/2} d\zeta\right] = \mathrm{const} = \frac{\pi H_0 d}{2}$$

$$= H_0\left[a E\left(\frac{(a^2 - b^2)^{1/2}}{a}\right) - \frac{b^2}{a} K\left(\frac{(a^2 - b^2)^{1/2}}{a}\right)\right] \tag{7}$$

where $E(q)$ and $K(q)$ are complete elliptic integrals.

At $|a-b| \ll a$ we obtain $(a^2-b^2)/a=\text{const}=d$. Consequently, the motion of the region of the reclosing flux has a constant velocity $\dot{a}=v_A(d/L)^{1/2}<v_A^*$.

4.Let us estimate the width of the tear d as the distance to which the pieces of the CS move apart during the time of the field screening. Within the limits of the applicability of the MHD description of the external plasma, this time is connected with the conductivity σ by the relation $\tau=4\pi\sigma d^2/c^2$. Substituting (4), we get

$$d = L/\mathrm{Re}_m^{2/3} \quad ; \qquad \mathrm{Re}_m = 4\pi\sigma v_A L/c^2. \tag{8}$$

Re_m is the magnetic Reynolds number. If $\mathrm{Re}_m \gg 1 (d \ll L)$, then the tearing mode stabilizes at the nonlinear level; for the CS to decay ($d>L$), we need an abrupt decrease of the electric conductivity of the external plasma, say as a result of the decrease of its concentration and of the development of plasma turbulence.

In relatively weak electric fields

$$(m_e/m_i)^2 > E^2/4\pi n_p T_p \approx \left(\frac{n_s T_s}{n_p T_p}\right)\left(\frac{\dot{a}}{c}\right)^2 \tag{9}$$

the electric-current density is limited by the buildup of the ion sound and is equal to $j=en_p v_{is}=en_p(T_p/m_i)^{1/2}$.[10] The dimension d is obtained from the equation $j=\text{curl}\, Hc/4\pi \approx H_0 c/4\pi d$

$$d = c\left(\frac{m_i n_s T_s}{4\pi n_p^2 e^2 T_p}\right)^{1/2} = \frac{c}{\omega_{pi}}\left(\frac{n_s T_s}{n_p T_p}\right)^{1/2}. \tag{10}$$

The tearing condition $(d>L=v_{Ti}c/\omega_{pi}u_i)$[11] is

$$\frac{d}{L} \approx \frac{u_i}{v_{Ti}}\left(\frac{n_s T_s}{n_p T_p}\right)^{1/2} > 1$$

(u_i is the directional velocity of the ions in the CS), i.e., at $n_s \approx n_p$ and $T_s \approx T_p$ it is equivalent to the requirement that the thickness of the layer be comparable with the ion Larmor radius. If a condition inverse to (9) is satisfied, then $j=en_p v_{Te} \times (E^2/4\pi n_p T_p)^{1/4}$ [10] and

$$d = \left[L\left(\frac{c}{\omega_{pi}}\right)^4 \frac{m_e}{m_i}\, \frac{n_s T_s}{n_p T_p}\left(\frac{c}{v_{Te}}\right)^2\right]^{1/5}. \tag{11}$$

For relations (9)–(11) to be satisfied we must have $\mathbf{j} \perp \mathbf{H}$, which is indeed the case in the tear.

We estimate in the vacuum approximation ($H^2 > 8\pi n_p m_e c^2$) the screening current near the null line ($\zeta=0$ on Fig. 1), where the particles are not magnetized in the region $|\zeta| < (m_e/m_i)^{1/6}(c^2Ed^2/eH_0^2)^{1/3}$ [12]; $E=H_0(\dot{a}/c)|_{a=d}$. If the ion Larmor radius exceeds this dimension, then

$$d = L\left(\frac{m_e}{m_i}\right)^{1/5}\left(\frac{n_s}{n_p}\right)^{3/5}\left(\frac{m_e c^2}{2\pi n_s L^2 e^2}\right)^{2/5} \tag{12}$$

5. Experiment[7] yielded $T_e \approx 30$ eV, $L \approx 0.6$ cm, $n_p \approx 2\times 10^{14}$ cm^{-3}, and $n_s \approx 8\times 10^{15}$ cm^{-3}. For expressions (10) and (11) we get $d \approx 7$ cm, which is comparable with the experimental value $d \approx 2$–3 cm. The velocity of the tear is higher than the Alfven velocity: in[7] they obtained $v_A \approx 5\times 10^6$ cm/sec and $\dot{a} \approx 10^7$ cm/sec.

1)It can be shown that during the linear stage the plasma outside the layer has a negligible effect on the tearing instability.

2)This can be assumed, since the velocity of the tear exceeds the Alfven velocity (see below).

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