Contribution to the theory of magnetosonic turbulence

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Strong turbulence of magnetosonic waves and the associated collisionless energy-dissipation mechanism are investigated.

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In the present article we consider magnetosonic-wave turbulence for which the principal nonlinear effect is modulation of the plasma density by the radiation pressure of the magnetosonic wave and the coupling, due to this modulation, of the magnetosonic wave with low-frequency quasineutral motions of the plasma. The initial system of equations for the description of the turbulence then takes the form

$$\left(\Delta-\frac{\omega_{pe}^2}{c^2}\right)\left[\left(1+\frac{\omega_{pe}^2}{\omega_{He}^2}\right)\frac{\partial^2}{\partial t^2}\Delta\phi+\omega_{pi}^2\Delta\phi+\omega_{pe}^2\frac{\partial^2\phi}{\partial z^2}-\frac{\omega_{pe}^4}{\omega_{He}^2}\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}\right]+\frac{\omega_{pe}^4}{c^2}\frac{\partial^2\phi}{\partial z^2}=\left(\Delta-\frac{\omega_{pe}^2}{c^2}\right)\frac{\omega_{pe}^2}{n_0\omega_{He}}\frac{\partial}{\partial t}[\nabla\delta n,\nabla\phi]_z; \qquad (1)$$

$$\frac{\partial}{\partial z}\left[\frac{\partial^2}{\partial t^2}-\frac{T_e+T_i}{M}\Delta\right]\delta n=-\frac{en_0}{M\omega_{He}}\Delta\,\overline{[\nabla v_z,\ \nabla\phi]_z}. \qquad (2)$$

In these equations $\phi(t,\mathbf{r})$ is the scalar potential in the magnetosonic wave, $\delta n(t,\mathbf{r})$ is the slow quasineutral variation of the density, the bar corresponds to averaging over the fast frequency, and v_c is the longitudinal velocity of the electrons:

$$m\frac{\partial v_z}{\partial t} = -eE_z.$$

The longitudinal electric field is connected with the scalar potential by the equation

$$(\omega_{pe}^2 - c^2\Delta)E_z = c^2\Delta\frac{\partial\phi}{\partial z}.$$

It is assumed in the initial system of equations that an inhomogeneity with respect to two mutually perpendicular axes is present in a plane perpendicular to the magnetic field; then the coupling between the low-frequency and high-frequency motions becomes anomalously strong (see[1,2]). The magnetosonic-wave dispersion law

$$\omega_k^2 = \omega_{LH}^2 \frac{k^2c^2 + \frac{M}{m}k_z^2c^2\frac{k^2c^2}{k^2c^2+\omega_{pe}^2}}{k^2c^2 + \frac{M}{m}\omega_{LH}^2\frac{\omega_{pe}^2}{\omega_{He}^2}} \qquad (3$$

($\omega_{LH}=\omega_{pi}/\sqrt{1+\omega_{pe}^2/\omega_{He}^2}$ is the frequency of the lower hybrid resonance) admits of the possibility of the localization of the waves in cavitons–regions of decreased density, from which the plasma is forced out by the radiation-pressure force. Cavitons can be strongly elongated along the magnetic field, $k_z/k \lesssim \sqrt{m/M}$. An important feature of the considered turbulence, which follows already from the dispersion law (3), is that the maximum variation of the plasma density and correspondingly the maximum field amplitude in the collapsing ($k\to\infty$) caviton is bounded:

$$\delta n \lesssim \frac{\omega_{LH} - \omega_k}{\partial\omega_{LH}/\partial n}.$$

Under these conditions collapse is impossible, and the short-wave transfer of the magnetosonic waves is due only to modulation instability. The oscillations are transferred into the region of shorter wavelengths

$$k \gtrsim k_* = \sqrt{\frac{\omega_{pe}}{cR}},$$

$$R = \begin{cases} \sqrt{3}\quad v_{Ti}/\omega_{pi}, & \text{if}\quad \omega_{He} >> \omega_{pe}, \\ \sqrt{\frac{3}{4} + 3\frac{T_i}{T_e}}\quad v_{Te}/\omega_{He}, & \text{if}\quad \omega_{He} << \omega_{pe}, \end{cases} \qquad (4$$

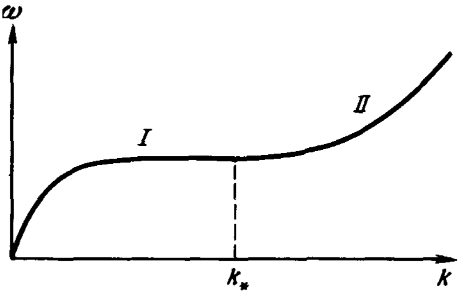

FIG. 1. I—Region of magnetosonic oscillations (step-like transfer), II—region of lower hybrid resonance (collapse and absorption by the particles).

vhere the considered branch of the magnetosonic waves "joins" the electrostatic ranch of the lower hybrid resonance (see Fig. 1). The dispersion of the latter is due o the thermal motion $\omega_k^2=\omega_{LH}^2+k^2R^2$ and admits of the existence of collapse. A heory for the dissipation due to absorption of waves by particles in collapsing cavi-ons, in the region of the lower hybrid resonance, was constructed in[2]. We examine in reater detail the short-wave transfer of magnetosonic waves under modulation insta-ility. The dispersion equation that describes the modulation instability of a pump vith frequency ω_{k_0}, a wave vector $\mathbf{k}_0$ and an amplitude E_0 can be obtained in accor-lance with the standard scheme of the linear theory. It takes the form

$$\omega^2 = k^2 c_s^2 \left\{ 1 - \frac{E_0^2}{2\pi n_0 T} \frac{[\mathbf{k}\mathbf{k}_0]_z^2}{k^2 |\mathbf{k}+\mathbf{k}_0|^2} \frac{1}{\omega_{LH}^2} \left(\frac{1}{k_0^2 c^2} - \frac{1}{|\mathbf{k}+\mathbf{k}_0|^2 c^2} \right) \right\}^{-1}, \tag{5}$$

$$T = T_e + T_i, \qquad c_s^2 = \{ T/M \}. \tag{6}$$

For simplicity we have written down the dispersion equation for perturbations with $ck \gg \omega_{pe}$ and $k_z = 0$. At pumps that are not too strong

$$\epsilon = \frac{E_0^2}{4\pi n_0 T} \frac{M}{m} \frac{\omega_{pe}^2}{\omega_{He}^2} << 1$$

he modulation instability produces primarily isotropization of the pump spectrum in ι plane perpendicular to the magnetic field. Transfer with respect to the modulus of he wave vector takes place in a sufficiently narrow frequency interval $\Delta_0 = |\omega_{\mathbf{k}+\mathbf{k}_0} \omega_{\mathbf{k}_0}| \sim \epsilon\omega_{LH}$ in the vicinity of the pump frequency. Under these conditions, the region of ower hybrid resonance is reached via multistep stage-by-stage transfer.

We obtain an equation for the stepwise transfer in an approximation where the nagnetosonic waves have random phases. The phase randomization mechanism is cattering by the density fluctuations produced by the low-frequency motions of the plasma. The magnetosonic-wave steady-state spectrum that results from the steplike transfer is then determined from the integral equation

$$\int d\mathbf{k}' \frac{k^2 k'^2 \sin^2\phi}{k^2 + 2kk'\cos\phi} \frac{|E_{k'}|^2}{16\pi^4 n_0 T} = \frac{\omega_{LH}^2}{c^2} . \tag{7}$$

Just as in the derivation of (5), we consider here perturbations with $kc \gg \omega_{pe}$, and $k_z = 0$; ϕ is the angle between the vectors **k** and **k**′. We consider a region of frequencies far enough from the pump, when the contribution of the latter to (7) can be neglected. Assuming isotropy of the spectrum, the approximate solution of this equation is

$$\frac{|E_{k_\perp}|^2}{4\pi n_0 T} = \frac{\omega_{LH}^2}{2\pi c^2 k_\perp^4} \ln^{-1} \frac{k_\perp c}{\omega_{pe}} . \tag{8}$$

This spectrum corresponds to an approximate (with logarithmic accuracy) equipartition of the energy of the magnetosonic waves over the frequencies, such that the energy directly connected with the pump in the frequency interval Δ_0 is $\approx E_0^2$. The total energy of the magnetosonic waves is

$$W = \int_{\omega_{pe}/c}^{k_*} \omega \frac{\partial \epsilon}{\partial \omega} |E_{k_\perp}|^2 k_\perp dk_\perp = n_0 T \frac{m}{M} . \tag{9}$$

The spectrum is maintained stationary by the pump, and in the solution there exists a constant energy flux towards smaller scales $J = \gamma_{\rm mod} W$ [$\gamma_{\rm mod} \sim k_0 c_s$ is the growth rate of the modulation instability of the pump and is determined from (5)]. The "effective collision frequency," which determines the rate of energy dissipation from the pump wave, turns out to be

$$\nu_{eff} = \frac{1}{E_0^2} \frac{dE_0^2}{dt} = \gamma_{\rm mod} . \tag{10}$$

The energy flux reaches the region of the lower hybrid resonance, where it is dissipated on account of collapse. The oscillation energy in this region can be estimated from the condition that the energy flux $J = -\nu_{\rm eff}^{LH} W^{LH}$ be constant over the spectrum, where

$$\nu_{eff}^{LH} = \omega_{LH} \frac{M}{m} \frac{\omega_{He}^2}{\omega_{pe}^2} \frac{W^{LH}}{4\pi n_0 T}$$

s the effective dissipation frequency for the lower hybrid resonance (see[2]). We thus obtain

$$W^{LH} = 4\pi n_0 T \frac{m}{M} \frac{\omega_s}{\omega_{LH}} \left[\frac{E_0^2}{4\pi n_0 T} \frac{M}{m} \frac{\omega_{pe}^2}{\omega_{He}^2} \frac{\omega_{LH}}{\omega_s} \right]^{1/2} .$$

$$\omega_s = k_0 c_s .$$

The factor in the square brackets is usually large in comparison with unity, in agreement with the condition for the applicability of the strong-turbulence approach to our problem. Integrating in (8) over $k \geqslant k_*$ and equating the result to W^{LH}, we verify that an oscillation accumulation takes place in the lower hybrid region and is due to the rather slow dissipation of the corresponding modes.

One of the possible applications of the reported results is a theory for collisionless shock wave propagating in a plasma transversely to a magnetic field.[3] The short-wave enhancement of the spectrum, due to the modulation instability, and the subsequent absorption by the particles, produce an effective mechanism for the dissipation of the energy of a nonlinear magnetosonic wave, and the threshold of this mechanism is much lower than that of current instabilities on the wavefront. The structure of the resultant shock wave is determined by the corresponding formulas of[3], in which the frequency of the pair collisions $\bar{\nu}$ is replaced by the effective dissipation frequency ν_{eff} determined by formula (10).

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